

Unit 1 Review

Points of Concurrency Graphic Organizer

| Segment Name | Definition | Point of Concurrency | Sketch of Point | Point Location | Key Characteristic |
|------------------------------------|---|--------------------------|-----------------|--|--|
| M Median | A line joining a vertex to the <u>midpoint</u> of the opposite side | C Centroid | | Always <u>inside</u> the triangle | The distance from a vertex of the triangle to the centroid is $\frac{2}{3}$ the length of the median |
| A Angle Bisector | A line which cuts an angle into <u>2 equal parts</u> | I Incenter | | Always <u>inside</u> the triangle | Incenter is <u>equidistant</u> from each side of the triangle |
| P Perpendicular Bisector | Perpendicular line through each side's <u>midpoint</u> | C Circumcenter | | Acute: <u>inside</u> the triangle Right: the <u>midpoint</u> of the hypotenuse Obtuse: <u>outside</u> the triangle | Circumcenter is <u>equidistant</u> from each vertex of the triangle |
| A Altitude | A <u>perpendicular</u> line from each vertex of the triangle to the opposite side | O Orthocenter | | Acute: <u>inside</u> the Triangle Right: on <u>the vertex</u> of the right angle Obtuse: <u>outside</u> the triangle | |

Parallelogram Properties

Remember, you have to be able to prove these properties using coordinates, congruent triangles, or parallel lines and apply them to solve problems!

All parallelograms have:

Opposite sides parallel and congruent/equal

Opposite angles equal/congruent

Diagonals that bisect each other

Diagonals that divide the parallelogram into ^{two} congruent triangles

Formulas to help: Slope = $\frac{y_2 - y_1}{x_2 - x_1}$

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Volume Formulas

Volume of a right prism (including a rectangular prism) or cylinder = Area of base • height

Volume of a cone or pyramid = $\frac{\text{Area of base} \cdot \text{height}}{3}$

Volume of a sphere = $\frac{4\pi r^3}{3}$

When using area or volume to calculate density, divide what you are measuring by the area or volume to determine the density per square (area) or cubic (volume) unit.

When using formulas to maximize volume, you can set up the volume formulas using the given parameters and use technology to find the maximum value. The steps in a TI-84 are:

1. Graph the formula in Y =
2. Adjust the window to see the minimum or maximum value for the appropriate domain
3. Push 2nd - Trace (Calc) - #3 minimum or #4 maximum, depending on the question asked
4. Set your bounds to the right and left of the minimum or maximum point, then get your value

Concept Questions:

1. What could the slope, midpoint, or distance formulas tell us about triangles or parallelograms?

Triangle - if it is a right triangle (perpendicular slopes) or isosceles/equilateral (distance/length of sides equal)

Parallelogram - opposite sides equal, diagonals bisect each other

2. What are some real-world applications of density? Why are they important?

Sample Population - how many people live in each square mile

Answers: Printer Quality - how many pixels per square inch

Chemicals in water - how many molecules per gallon

Unit 2 Review

Equation of a Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(h, k) = \underline{\text{center}} \qquad r = \underline{\text{radius}}$$

Example: Find the equation, center, and radius of the circle:

$$x^2 + y^2 + 16x - 6y = 8$$

Group x and y $(x^2 + 16x) + (y^2 - 6y) = 8$

Complete the Square $(\frac{b}{2})^2$ $(x^2 + 16x + 64) + (y^2 - 6y + 9) - 64 - 9 = 8$
Add Inside, Subtract outside

Factor and Combine $(x + 8)^2 + (y - 3)^2 - 73 = 8$

Write in Standard Form $(x + 8)^2 + (y - 3)^2 = 81$

Center = $(-8, 3) \rightarrow$ opposites of numbers in parentheses
 $r = 9 \rightarrow \sqrt{81}$

Circles

Area of a Circle = πr^2

Circumference of a Circle = πd or $2\pi r$

Area of a SECTOR of a circle = $\frac{\text{Degrees}}{360} \cdot \pi r^2$

Arc Length of a Sector of a circle = $\frac{\text{Degrees}}{360} \cdot \pi d$

Tangent lines, lines outside the circle that touch it at one point, form a right angle with the radius it intersects.

A radius that bisects a chord forms a right angle with the chord.

Radian Angle Measure

Radians = $\frac{\text{Degrees} \cdot \pi}{180}$

Degrees = $\frac{\text{Radians} \cdot 180}{\pi}$

Examples: Convert $\frac{5\pi}{4}$ radians to degrees.

$$\frac{5\pi}{4} \cdot \frac{180}{\pi} = \frac{900\pi}{4\pi} = 225^\circ$$

Convert 300° to radians.

$$\frac{300\pi}{180} = \frac{5\pi}{3} \text{ rad}$$

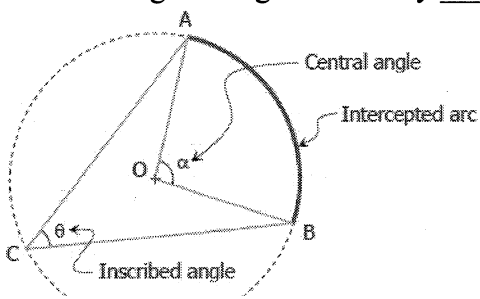
Radian Measure in Circles: Arc Length = radius \cdot radius

Angle-Arc Relationships

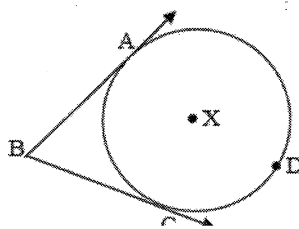
Central Angle - equal measure of intercepted arc, vertex at the center

Inscribed Angle - half the measure of intercepted arc, vertex on the circle

Circumscribed Angle - Angle formed by tangent lines to circle



Circumscribed Angle

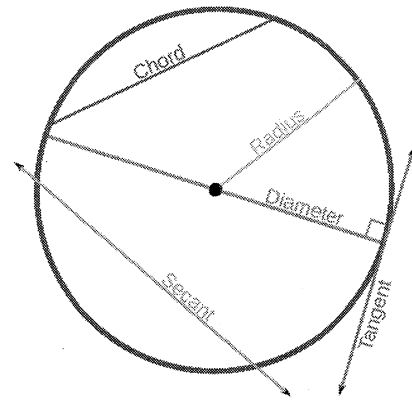


Circle Segments

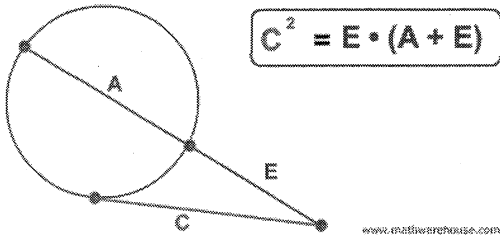
Chord - Segment connecting two points on the circle

Tangent - Segment that touches a circle in one point

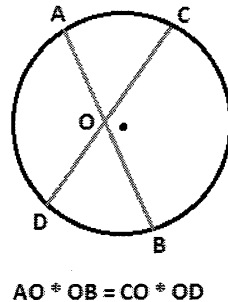
Secant - Line/segment that intersects a circle in two points



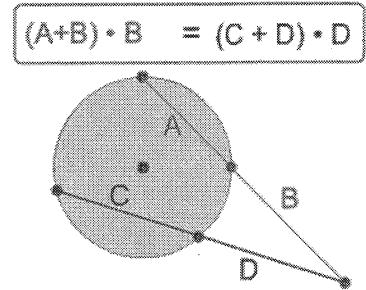
Secant-Tangent Intersect



Two Chords Intersect



Two Secants Intersect



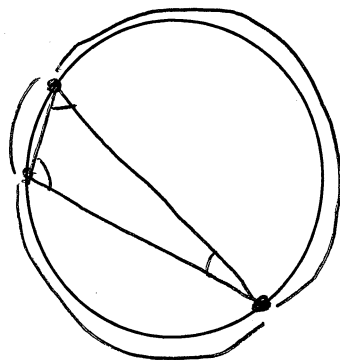
Conceptual Questions:

1. How does the Pythagorean Theorem relate to the equation of a circle?

~~The radius~~ The radius represents the distance from the center to every (x,y) point on the circle, and $(x-h)$ and $(y-k)$ represent the lengths of the sides. $a^2 + b^2 = c^2 \rightarrow (x-h)^2 + (y-k)^2 = r^2$

2. Draw a triangle inside the circle below with all its vertices on the circle. What kind of angles are the vertices?

Why is an inscribed angle half the measure of the intercepted arc?



Triangle Angles (180°) intercept entire circle (360°), so inscribed angles are half the arc measure (180 is half of 360)

3. Why do we divide the degrees by 360 to compute the arc length or area of a sector of a circle?

The arc length or area of the sector represents that fraction of the circle - $\frac{\text{Degrees}}{360}$

Unit 3 Review

Statistics

Experiment – Study comparing a control group and an experimental group

Observation – Study observing the characteristics of one group

Simple Random Sample – Subjects are chosen from a population completely randomly

Systematic Random Sample - Subjects are chosen based on some ^{order} ~~demographic data~~

Convenience Sample - Subjects are chosen by what is easiest

Stratified Random Sample - Subjects are chosen based on some demographic data

Mean (\bar{x}) – Statistical average

Standard Deviation (σ) – Amount which data varies from the mean

Margin of Error - Expected value that a sample mean could deviate from the actual population mean

Margin of Error Formula: $\frac{\text{Standard Deviation}}{\sqrt{n}}$ (where n is the sample size)

Bias - Unintended feelings or actions that skew data

Concept Questions:

1. Why is it important that bias is limited and samples are random in a statistical study?

To gain the most accurate, representative data for a population

2. What is more important in evaluating data, the mean or standard deviation? Why?

Answers will vary

Unit 4 Review

Definition of Inverse Functions

Inverse functions have the input (x) and output (y) values switched. Their graphs are a reflection over the $y=x$ line.

To find an inverse function, replace $f(x)$ with y and switch the x and y .

Example 1: Find the inverse of $f(x) = (x+4)^3 - 6$.

$$y = (x+4)^3 - 6$$

$$x + 4 = (y+6)^{1/3}$$

$$x = (y+6)^{1/3} - 4$$

$$f^{-1}(x) = \sqrt[3]{x+6} - 4$$

Example 2: Find the inverse of $f(x) = 4^{x-2} - 1$.

$$y = 4^{x-2} - 1$$

$$x + 1 = 4^{y-2}$$

$$\log_4(x+1) = y - 2$$

$$\frac{\log(x+1)}{\log 4} = y - 2$$

$$\frac{\log(x+1)}{\log 4} + 2 = y$$

$$f^{-1}(x) = \frac{\log(x+1)}{\log 4} + 2$$

Restricting the Domains to Create Inverse Functions

For a relation to be a function, every input (x) produces one output (y).

Sometimes, a function will produce an inverse that is not a function. In that case, the domain must be restricted so the inverse is also a function. One example of this is with quadratic functions, whose inverse is a square root function function.

Example 3: Find the inverse of $f(x) = x^2 - 1$, and determine the domain on which the inverse exists.

$$x = y^2 - 1$$

$$x + 1 = y^2$$

$$\sqrt{x+1} = y$$

$$f^{-1}(x) = \sqrt{x+1}$$

Domain - $x + 1 \geq 0$

$x \geq -1$

Inverses of Exponential Functions

Exponential functions, with a variable as the exponent, use an operation called logarithms to determine the inverse.

To convert exponents to logarithms:

$$b^x = a \rightarrow \log_b a = x$$

$$10^x = a \rightarrow \log a = x$$

$$e^x = a \rightarrow \ln a = x$$

Concept Questions:

1. How are inverse functions and inverse operations related? They both represent the opposite of a given operation that will negate the value.

2. Why can there never be a logarithm of a negative number? (Why, in the above examples, will a never = 0?)

An exponent cannot have a negative answer, so the inverse of an exponent (logarithm) cannot apply to a negative number.

Unit 5 Review

Exponential Functions

$$y = ab^x \text{ (Growth or Decay)}$$

$$A = Pe^{rt} \text{ (Growth compounding CONTINUOUSLY)}$$

$$y = \frac{\text{Final Amount}}{a = \text{Initial Amount}} t$$

$$A = \frac{\text{Final Amount}}{P = \text{Principal (Initial Amount)}} e^{rt}$$

$$b = \frac{\text{rate of growth/decay}}{x = \text{independent variable}} \quad e = e \quad r = \frac{\text{rate of growth}}{t = \text{independent variable}}$$

If a value increases or decreased by a percentage, the growth or decay factor is represented by $1+r$ or $1-r$, where r is the percent increasing or decreasing as a decimal.

Solving Exponential Equations for the Exponent

To solve exponential equations to determine the exponent:

1. If the bases are equal, set the exponents equal and solve.
2. If only one side has an exponent, isolate the base and exponent.
3. Convert the exponent to a logarithm (the inverse of an exponent).
4. Use the change of base formula if necessary to evaluate the logarithm.
5. If the variable is not isolated, finish solving the equation.

NOTE: Exponential equations, like all other equations, can be solved by graphing the expressions on both sides of the equation and finding the point of intersection using technology.

Change of Base Formula

$$\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$$

We can evaluate \log and \ln , the common logarithms, using the calculator to get a decimal approximation.

Graphs of Exponential Functions

$$y = ab^x \quad a = \text{y-intercept} \quad b = \frac{\text{rate of growth or } \overset{\text{decay}}{\text{decay}}}{\text{decay}}$$

For $b > 1$, graph increases. For $0 < b < 1$, graph decreases.

For increasing exponential functions, they will ultimately increase faster than other functions as x increases.

Concept Questions:

1. Why do we add or subtract 1 when determining the growth factor from a percent increase or decrease?

The percent increase or decrease is added or subtracted to the existing value, represented by 1 (100%).

2. Will an exponential decay function ever equal 0? Why or why not?

No, because when a value decays by a factor, there will always be some remaining.

Unit 6 Review

Absolute Value Functions

The absolute value of a number, represented $|x|$, represents its distance from zero on a number line.

It is always positive.

The graph of an absolute value function is in the shape of a \checkmark , because negative inputs in the domain have positive outputs. They follow the same transformation rules as other functions.

Translations:

| | Inside Function | Outside Function |
|--------------|-----------------|------------------|
| Positive (+) | left | up |
| Negative (-) | right | down |

Reflections (Flips): When the coefficient outside the bars is negative, graph reflects over y-axis

Stretches/Shrinks: If $a > 1$, graph narrows. If $a < 1$, graph widens.

Absolute value functions can be graphed in the calculator using MATH-NUM-abs(. To solve systems involving these equations, find the point of intersection with the other equation.

Step Functions

Greatest Integer Function - For any x-value, the y-value is the greatest integer less than or equal to x.

It can be graphed using the following steps: Press [Y=]. If anything is already entered, press [CLEAR].

Press [MATH]. Arrow to "NUM," then select 5: int(.

Enter the expression contained within the greatest integer function using x as the variable. Enter the remaining part of the expression by moving the cursor to the left and/or right of the int() expression. For example, "int($x+3$)" is equivalent to the expression $\lfloor x+3 \rfloor$.

Press [GRAPH].

Least Integer Function - For any x-value, the y-value is the least ^{greater} integer ~~less~~ than or equal to x.

It can be graphed using the following steps: Press [Y=]. If anything is already entered, press [CLEAR].

Press [MATH]. Arrow to "NUM," then select 5: int(.

Enter the expression contained within the least integer function, using x as the variable. Enter a negative sign before the entire expression in the least integer brackets. Enter a negative sign before the int(function. Enter the remaining part of the expression by moving the cursor to the left and/or right of the int() expression.

For example, " $-\text{int}(-(x+3))$ " is equivalent to the expression $\lceil x+3 \rceil$. Press [GRAPH].

Piecewise Functions

Piecewise functions are functions with different function rules for different domains.

$$\text{For example, they can be represented: } f(x) = \begin{cases} -2x, & \text{for } x \leq 0 \\ 2^x, & \text{for } 0 < x < 5 \\ x^2, & \text{for } x \geq 5 \end{cases}$$

The domain is usually continuous, but the range is not necessarily continuous depending on the values.

When evaluating piecewise functions, substitute the input into the appropriate rule for its domain.

When graphing piecewise functions, graph each function rule for its appropriate domain.

Use open circles for $<$ and $>$, and use closed circles for \leq and \geq .

Building New Functions (Operations With Functions)

Functions can be added, subtracted, multiplied, and divided to create new functions following the same rules that apply to other expressions.

The domain of the new functions can change, however, if the operation creates domain restrictions (such as 0 under a fraction bar or negatives under a radical) in the new function.

Examples:

For $f(x) = 3x + 6$ and $g(x) = x + 2$,

a) What is $f(x) + g(x)$? What is its domain?

$$(3x+6) + (x+2) = 4x+8 \quad \text{All real numbers}$$

b) What is $f(x)/g(x)$? What is its domain?

$$\frac{3x+6}{x+2} = \frac{3(x+2)}{x+2} = 3 \quad \text{Domain - All real numbers } \neq -2, \text{ because } -2 \text{ would give 0 under fraction bar}$$

Concept Questions:

1. How is an absolute value function the same as a piecewise function?

An absolute value function is a piecewise function of 2 linear functions with opposite slopes.

2. Consider the functions: $f(x) = 4x + 9$ and $g(x) = -2x - 4$

$$\text{Evaluate } f(-3). \quad 4(-3) + 9 = -3$$

$$\text{Evaluate } g(-3). \quad -2(-3) - 4 = 2$$

$$\text{Add } f(x) + g(x). \quad (4x+9) + (-2x-4) = 2x+5$$

$$\text{Evaluate } (f+g)(-3). \quad 2(-3) + 5 = -1$$

What do you notice? What properties have you learned that explain your answer?

$$f(-3) + g(-3) = (f+g)(-3)$$

$$-3 + 2 = -1 \quad \checkmark$$

The commutative and associative properties apply to make this true.

Unit 7 Review

Key Terms

Degree - highest exponent in a polynomial

Leading Coefficient - coefficient of the term with the highest polynomial

Solution - x-values when a polynomial equals zero

Graphs of Polynomials

Graphs of polynomials follow many of the same patterns as other graphs.

x-intercepts (also roots, solutions, zeroes): where $y=0$, graph touches x-axis

y-intercepts: where $x=0$, graph touches y-axis

Relative Minimum/Maximum Values: where graph changes direction, can be found using technology -

2^{nd} -TRACE (Calc), #3 Minimum or #4 Maximum Set Bounds

End Behavior as x approaches ∞ and $-\infty$:

| | Odd Degree (Like a line) | Even Degree (Like a parabola) |
|------------------------------|---|---|
| Positive Leading Coefficient | As $x \rightarrow -\infty$, y <u>decreases</u> As $x \rightarrow \infty$, y <u>increases</u> | As $x \rightarrow -\infty$, y <u>increases</u> As $x \rightarrow \infty$, y <u>increases</u> |
| Negative Leading Coefficient | As $x \rightarrow -\infty$, y <u>increases</u> As $x \rightarrow \infty$, y <u>decreases</u> | As $x \rightarrow -\infty$, y <u>decreases</u> As $x \rightarrow \infty$, y <u>decreases</u> |

Dividing Polynomials

Long Division - To divide any polynomial by another polynomial.

Synthetic Division - Shortcut to divide a polynomial by a binomial (with leading coefficient of 1)

Polynomial Long Division

1. x (ignore the "+3") goes into x^2 x^2 times. Put the x^2 on top of the x^2 . (Some teach to put it on top of the $7x^2$; it doesn't really matter).
2. Multiply the x^2 by " $x+3$ " to get x^3+3x^2 , and put it under the x^3+7x^2 . (Always line up the terms).
3. Subtract down, and bring the next term (+10x) down.
4. x (ignore the "+3") goes into $4x^2+10x$ 4x times. Put the 4x on top of the $7x^2$.
5. Multiply 4x by " $x+3$ " to get $4x^2+12x$, and put it under the $4x^2+10x$.
6. Subtract down, and bring the next term (-6) down.
7. x (ignore the "+3") goes into $-2x-6$ -2 times. Put the -2 on top of the 10x.
8. Multiply -2 by " $x+3$ " to get $-2x-6$, and put it under the $-2x-6$.
9. Subtract down; there is no remainder.

$$\begin{array}{r} \text{Divide} \\ x^3+7x^2+10x-6 \\ x+3 \end{array}$$

$$\begin{array}{r} x^2+4x-2 \\ (+3)x^3+7x^2+10x-6 \\ \underline{x^3+3x^2} \\ 4x^2+10x \\ \underline{4x^2+12x} \\ -2x-6 \\ \underline{-2x-6} \\ 0 \end{array}$$

Synthetic Division

$$\text{Divide } \frac{x^3+7x^2+10x-6}{x+3}$$

$$\begin{array}{r|rrrr} -3 & 1 & 7 & 10 & -6 \\ & & -3 & -12 & 6 \\ \hline & 1 & 4 & -2 & 0 \end{array}$$

Go down a level (subtract 1) with the exponents for the variables:

$$\begin{aligned} 1x^2+4x-2 \\ = x^2+4x-2 \end{aligned}$$

1. Take the bottom (the divisor) and set to 0 and solve for x. (This may or may not be a factor, depending on whether our remainder is 0.) In our case, we get -3. Put a "corner" around the -3.
2. Take the coefficients of the polynomial on top (the dividend) put them in order from **highest** exponent to **lowest** and put them next to the -3. **If there is a term missing, you have to include a 0 for that term.** For example, for $3x^2-2$, you'd have to put "3 0 -2" (0 for the x^2 that is missing).
3. Bring the first coefficient (1) down.
4. Multiply the -3 by the 1 on the bottom and put the product (-3) under the 7. Add down to get 4.
5. Multiply the -3 by the 4 on the bottom and put the product (-12) under the 10. Add down to get -2.
6. Continue with this pattern until you get to the end of the coefficients. The last number in the bottom right corner is the **remainder**. We got a remainder of 0; this means that $x+3$ goes into $x^3+7x^2+10x-6$ exactly, so it is a **factor**.
7. To get the quotient, use the numbers you got up until the remainder as coefficients, but subtract 1 for each of the terms' exponents.

Remainder Theorem - When a polynomial is divided by a binomial $(x - k)$, the remainder is equal to the y-value at $f(k)$.

Factor Theorem - When a polynomial is divided by a binomial $(x - k)$ and the remainder is 0, $(x - k)$ is a factor of the polynomial. Therefore, the solution for x to the equation $x - k = 0$ is a root of the polynomial.

Fundamental Theorem of Algebra

Fundamental Theorem of Algebra - The number of solutions, real or complex, to any function is equal to its degree.

Building Polynomials from Roots

If we know the solutions to polynomial functions, we can write polynomials equal to 0 that associate with the roots. Then, using these points and one more point, we can write the function.

Example: What function has roots -2, 4, and 5 and passes through point (1, 9).

Write Equation Using Factors $y = a(x+2)(x-4)(x-5)$

Substitute other point

$$9 = a(1+2)(1-4)(1-5)$$

$$9 = a(3)(-3)(-4)$$

$$9 = a(36)$$

$$\frac{1}{4} = a$$

$$y = \frac{1}{4}(x+2)(x-4)(x-5)$$

$$y = \frac{1}{4}(x^2 - 2x - 8)(x - 5)$$

$$y = \frac{1}{4}(x^3 - 7x^2 - 18x + 40)$$

$$y = \frac{1}{4}x^3 - \frac{7}{4}x^2 - \frac{9}{2}x + 10$$

We can also use the calculator's regression feature to build these functions. Using the example above:

1. STAT-EDIT, use points $(-2, 0)$, $(4, 0)$, $(5, 0)$, $(1, 9)$

2. STAT-CALC-CubicReg (because the function has 3 solutions)

3. Substitute the coefficients to get the function: $y = 0.25x^3 - 1.75x^2 - 4.5x + 10$

Solving Systems With Polynomials

To solve systems of equations with polynomials and other functions, using technology to find the points of intersection is usually the most efficient way. Graph both equations on the same coordinate plan, and determine what points satisfy both equations.

Concept Questions:

1. How do lines and parabolas relate to the end behavior of all polynomial functions?

Lines have an odd degree, and parabolas have an even degree. So the behaviors of:

$$y = x \quad y = -x \quad y = x^2 \quad y = -x^2 \quad \text{represent all polynomials.}$$

2. Why does the Remainder Theorem guarantee that dividing a polynomial by a binomial to produce a remainder of zero proves that the solution to the binomial is a solution to the polynomial?

A remainder of 0 means that the y value at that point is 0, so the number has to be an x-intercept and solution.

Unit 8 Review

Operations With Rational Expressions

Rational expressions follow the same arithmetic rules as fractions.

Multiply rationals - Factor first, then multiply across, then divide out common factors

Dividing rationals - Factor first, then multiply times the reciprocal, then divide out common factors

Adding or subtracting rationals - Factor denominator first, then determine common denominator. Multiply numerators and denominators to get a common denominator, then add or subtract the numerators.

Solving Rational Equations

FACTOR FIRST!!! Find common denominator for all terms, and multiply numerator

AND denominator to get common denominator. Then, multiply to cancel all

denominators and set numerators equal to solve. Don't forget to

check domain restrictions where denominator = 0

Graphing Rational Functions (Factor First!):

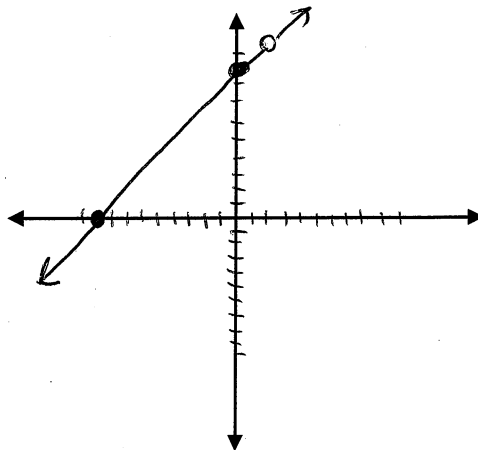
Vertical Asymptotes/Holes - Set denominator = 0 (Factors that cancel are holes, factors that remain in denominator are vertical asymptotes)

Horizontal Asymptotes: Degree of Numerator Higher - None Degree of Denominator Higher - 0

Degree of Numerator and Denominator Equal - Divide leading coefficients

x-intercepts - Set Numerator = 0 after cancel y-intercepts - Set x = 0 and evaluate

Example: $f(x) = \frac{x^2 + 7x - 18}{x - 2} = \frac{(x+9)(x-2)}{x-2}$



VA - None

Holes - $x - 2 = 0 \rightarrow x = 2$

HA - None

x-int: $x + 9 = 0$
 $x = -9$

y-int: $\frac{0^2 + 7(0) - 18}{0 - 2} = 9$

Concept Questions:

1. Why can we "cancel out" common factors in the numerator and denominator, and why is "cancel out" not completely accurate? Common factors divide to equal 1, and multiplying by 1 does not affect the value. The factors do not cancel, or go away - they divide to 1.

2. Why do vertical asymptotes and holes exist on a rational function graph where the denominator = 0?

There can be no value for a function where the denominator = 0, and this is represented by vertical asymptotes and holes.

Unit 9 Review

| Trigonometric Function | Abbreviation | Ratio of Sides in Right Triangle | Unit Circle Coordinate | Possible Value | x-intercepts on graph | y-intercept on graph |
|------------------------|--------------|---|------------------------|---------------------------------|--|----------------------|
| Cosine | \cos | $\frac{\text{adjacent leg}}{\text{hypotenuse}}$ | x | $0 \leq x \leq 1$ | $90^\circ, 270^\circ, \dots$ $\frac{\pi}{2}, \frac{3\pi}{2}, \dots$ | 1 |
| Sine | \sin | $\frac{\text{opposite leg}}{\text{hypotenuse}}$ | y | $0 \leq y \leq 1$ | $0^\circ, 180^\circ, 360^\circ, \dots$ $0, \pi, 2\pi, \dots$ | 0 |
| Tangent | \tan | $\frac{\text{opposite leg}}{\text{adjacent leg}}$ | $\frac{y}{x}$ | $-\infty \leq \tan \leq \infty$ | $0, 180^\circ, 360^\circ, \dots$ $0, \pi, 2\pi, \dots$ | 0 |

Measuring Angles on the Coordinate Plane

Angles are measured in a circle on the coordinate plane, starting at the initial side, the positive x-axis, and going to the terminal side, the other ray of the angle. The angles are measured in a counterclockwise direction.

A 90° angle has its terminal side on the positive y-axis, and 180° angle has its terminal side of the negative x-axis, and a 270° angle has its terminal side on the negative y-axis. The angle measure keeps increasing as the terminal side continues counterclockwise, even beyond 360° .

Angles can be measured in degrees or radians, with 2π radians measuring the same as 360° , or a circle.

The Unit Circle

The unit circle is a circle on the coordinate plane with its center at the origin $(0,0)$ and a radius of 1.

The key points on the unit circle are determined by $30-60-90$ and $45-45-90$ special right triangles, and the trig values of these angles represent the coordinates of the points.

Sine and Cosine Graphs

The trigonometric ratios are functions of the angles they associate with, so they can be graphed as functions on the coordinate plane. The angle is the x-axis, and the ratio is the y-axis.

The graphs are cyclical, as they repeat the same pattern.

Concept Questions:

1. Why are trigonometric graphs cyclical, based on the unit circle?

The angle measures continue around the circle to infinity, following the same pattern of trig ratios.

2. How do right triangle trigonometric ratios relate to the coordinate plane trigonometric ratios on the unit circle?

The sides of the special right triangles are divided based on the ratios to relate the ratios, angles, and coordinate points.