Model Mathematics ACT II

60 Questions—60 Minutes

INSTRUCTIONS: Find the solution to each problem, choose the correct answer choice, then darken the appropriate oval on your answer sheet. Do not spend too much time on any one problem. Answer as many problems as you can easily and then work on the remaining problems within the time limit for this test. Check pages 417–424 for answers and explanations.

Unless the problem indicates otherwise:

- figures are NOT necessarily drawn to scale
- geometric figures are plane figures
- a *line* is a straight line
- an *average* is the arithmetic mean

You may use a calculator.

1. $\triangle ABC$ is an isosceles triangle with the measure of $\angle C = 110°$. What is the measure of $\angle B$?
   A. 35°
   B. 45°
   C. 70°
   D. 90°
   E. 120°

2. Mary has saved $160 of the $250 she needs for rent. She takes home $6 an hour from her part-time job. Mary works the same number of hours each day for 3 days. How many hours does Mary have to work each day to take home enough money to pay the balance of her rent?
   F. 3 hours
   G. 4 hours
   H. 5 hours
   J. 6 hours
   K. 7 hours

3. Basil earns $6 an hour when he works up to 8 hours in a day. Basil earns $9 an hour for each hour over 8 hours a day that he works. If Basil earned $84 on Tuesday, how many hours did he work that day?
   A. 4
   B. 6
   C. 8
   D. 12
   E. 16

4. In the equation $d = rt$, where $d =$ distance, $r =$ rate, and $t =$ time. If you travel a distance of 260 miles in 4 hours, at what rate are you traveling?
   F. 45 mph
   G. 60 mph
   H. 65 mph
   J. 70 mph
   K. 85 mph

5. In the figure below, lines p and ℓ are parallel. Line t is a transversal that crosses ℓ and p but is not perpendicular to them. Which pair of angles is not supplementary?
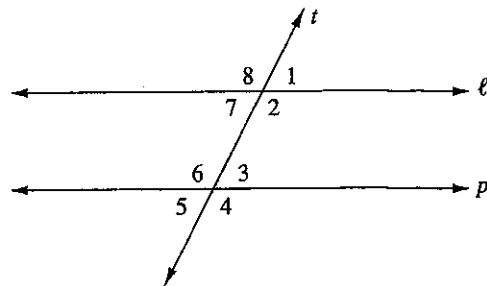

   A. $\angle 1$ and $\angle 8$
   B. $\angle 2$ and $\angle 3$
   C. $\angle 4$ and $\angle 5$
   D. $\angle 3$ and $\angle 5$
   E. $\angle 6$ and $\angle 7$

GO ON TO THE NEXT PAGE.

6. In scientific notation $15,000 \times 2,500 = ?$

   F. 3.75×10^{-7}
   G. 6×10^0
   H. 1.75×10^4
   J. 4.75×10^6
   K. 3.75×10^7

7. If $0.04 \times n = 3.626$, then $n = ?$

   A. 0.09065
   B. 9.065
   C. 90.65
   D. 906.5
   E. 9,065

8. When $x \neq 3, \dfrac{x^2 - 9}{x - 3} = ?$

   F. $(x - 3)(x + 3)$
   G. $(x + 3)$
   H. $(x + 9)$
   J. $(x + 12)$
   K. $(x + 15)$

9. What is the value of the expression $3y^2 - 2y + 3$ for $y = 3$?

   A. 18
   B. 21
   C. 24
   D. 26
   E. 36

10. A sweater is on sale for $38.25, after a 15% reduction from the original price. What was the original price of the sweater?

    F. $32.50
    G. $40.00
    H. $44.25
    J. $45.00
    K. $50.00

11. For all x, $(x - 4)(x^2 + 4x + 16) = ?$

    A. $8x^2 + 4x + 24$
    B. $x^3 - 64$
    C. $(x + 8)^2$
    D. $(x + 4)^3$
    E. $4(x + 4)^2$

12. If a car travels 75 miles in 2 hours, how far will the car travel at the same rate in 5.5 hours?

    F. 150 miles
    G. 200 miles
    H. 206.25 miles
    J. 400 miles
    K. 412.5 miles

13. In the diagram below, line segment $\overline{AB}$ is 8 units long and line segment $\overline{CD}$ is 6 units long. The two segments are perpendicular bisectors of each other. Another line segment, $\overline{DB}$, joins the endpoints of segments $\overline{AB}$ and $\overline{CD}$. How long is line segment $\overline{DB}$?
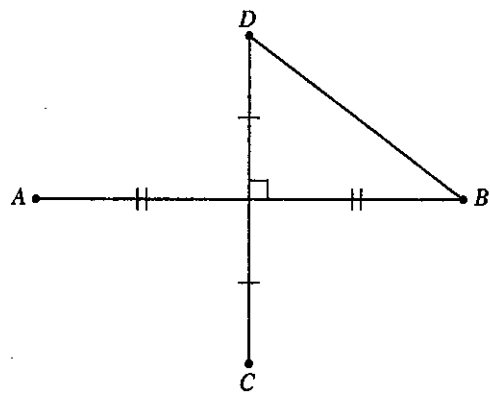

    A. 3
    B. 4
    C. 5
    D. 6
    E. 7

14. A floor in the shape of a triangle has an area of 24 square feet. If the base is 2 feet less than the height, which equation would be used to find the height of the triangle?

    F. $48 = (h + 2)h$
    G. $24 = (h + 2)h$
    H. $48 = h^2$
    J. $48 = (h - 2)h$
    K. $48 = (h - 2)h + 2h$

15. In Joan's bank account, which pays simple interest, $2,500 would grow at a fixed rate to $3,125 in 5 years. If she were to place $5,000 in this account, how much interest would she earn after 7 years?

    A. $1,750
    B. $6,250
    C. $7,000
    D. $17,500
    E. $28,000

16. A window company produces a rectangular window that is 12 feet high and 8 feet wide. They also produce a square window that has the same area as the rectangular window. How long, in feet, is each side of the square window?

    F. 8
    G. $\sqrt{6}$
    H. $4\sqrt{6}$
    J. 4
    K. 16

GO ON TO THE NEXT PAGE.

17. A video store has 20 copies of a movie that each rent for $3 a day. If 75% of these copies are rented on one day, and 90% of these copies are rented the next day, how much money was collected from renting this movie on both days?

A. $45
B. $54
C. $85
D. $99
E. $100

18. In the figure below, line ℓ and line m are parallel. Line t is a transversal that crosses both lines ℓ and m. Given that $\angle 1$ is 35°, what is the measure of $\triangle BAD$?
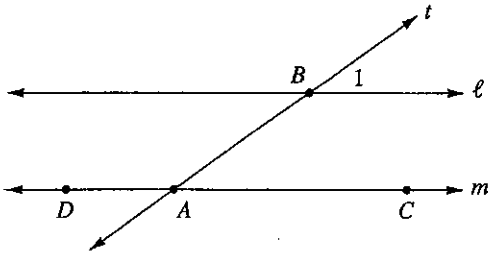

F. 35°
G. 55°
H. 135°
J. 145°
K. 215°

19. What is the value of x in the following system of equations?

$2x + 3y = 4$
$9x + 7.5y = 18$

A. 1
B. 2
C. 3
D. 4
E. 5

20. $\left(\frac{3}{5} + \frac{3}{15}\right) \times 5 - \left(\frac{3}{4} \div \frac{1}{4}\right) = ?$

F. 0
G. $\frac{1}{4}$
H. $\frac{1}{2}$
J. 1
K. 3

21. Which of the following expressions is equivalent to $-(3x + 5) - 2x + 2x(2x + 3)$?

A. $-4x^2 + x + 5$
B. $15x - 5$
C. $4x^2 - 5x$
D. $-4x^2 + 11x - 2$
E. $4x^2 + x - 5$

22. Simplify the following expression: $\frac{c^2 + 6c + 9}{2c + 6}$. $(c \neq -3)$

F. $(c + 3)$
G. $(c - 3)$
H. $\frac{(c + 3)}{2}$
J. $\frac{(c - 3)}{2}$
K. $\frac{(c + 3)^2}{2}$

23. If $x = 7$ in the equation $3y + 4 - 2x = 0$, what is the value of y?

A. $\frac{1}{3}$
B. $\frac{2}{3}$
C. 3
D. $3\frac{1}{3}$
E. 6

24. In $\triangle ABC$, segment $\overline{BD}$ is perpendicular to segment $\overline{AC}$. If the measure of $\angle DAB$ is 25°, and the measure of $\angle ABC$ is 120°, what is the measure of $\angle DBC$?
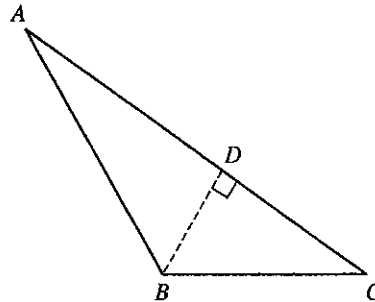

F. 40°
G. 45°
H. 55°
J. 65°
K. 90°

25. Six runners begin a race. If all the runners finish and there are no ties, in how many different orders can the runners cross the finish line?

A. 8,776
B. 720
C. 36
D. 30
E. 20

GO ON TO THE NEXT PAGE.

26. For all positive values m, n, and p, with $m > n > p$, which of the following statements is always true?

F. $m + n < p$
G. $2p + m > n$
H. $p + n < m$
J. $2n - m > p$
K. $2m + n < p$

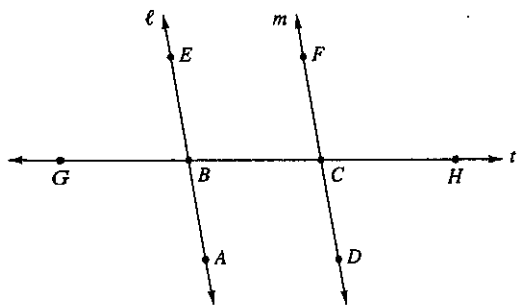**27.** In the right triangle shown, cosine $\theta = \dfrac{6}{10}$. How long is leg b?
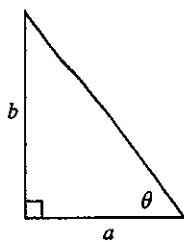
A. 4
B. 8
C. 9
D. 10
E. 12

28. Find the value(s) of y in the following equation:

$$\frac{2y + 3}{\frac{y}{3} + 2} = \frac{3y}{6 + y}.$$

F. $\{-3\}$
G. $\{3, 6\}$
H. $\{3\}$
J. $\{-6\}$
K. $\{-3, -6\}$

29. In the figure shown, lines ℓ and m are parallel, and line t is a transversal that crosses both lines ℓ and m. If the measure of $\angle ABC$ is 30°, what is the measure of $\angle FCH$?

A. 30°
B. 45°
C. 90°
D. 120°
E. 150°

30. $2|-8| + 3|-6| - |-2| = ?$

F. -32
G. -30
H. 8
J. 30
K. 32

31. If $x - 6 = -x^2$, find the possible value(s) of x.

A. $\{-2, -3\}$
B. $\{2, 3\}$
C. $\{2, -3\}$
D. $\{4, 6\}$
E. $\{-4, -6\}$

32. A carpenter cuts a rectangular piece of plywood in half diagonally to make two congruent triangles. If the width of the rectangle is 4 feet, and the cut along the diagonal is $4\sqrt{5}$ feet, what is the length of the rectangle?

F. 6 feet
G. $5\sqrt{4}$ feet
H. 5 feet
J. 8 feet
K. $3\sqrt{5}$ feet

33. What is the radius of a circle whose equation is $(x - 2)^2 + (y + 1)^2 = 54$?

A. $3\sqrt{5}$
B. 2
C. $3\sqrt{6}$
D. 8
E. $2\sqrt{5}$

34. In slope-intercept form, what is the equation of a line having a slope of -2 and a y-intercept of 5?

F. $y = -2x + 5$
G. $y = 5x - 2$
H. $2x + y + 5 = 0$
J. $5x - 2y + 1 = 0$
K. $y = 3x + 5$

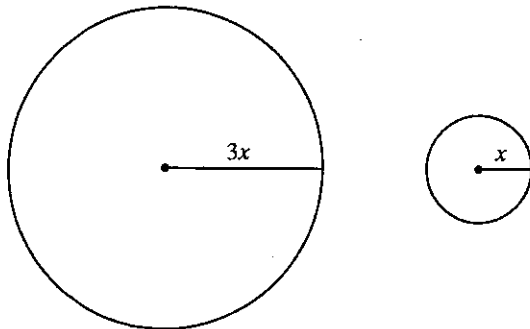**35.** The scores on a math test are shown in the chart below. What is the average score on this test?

Score	No. of Students
90	4
85	6
80	2
75	1

A. 90
B. 85
C. 82.5
D. 80
E. 75

GO ON TO THE NEXT PAGE.

36. In the two circles shown below, the radius of the larger circle is three times the radius of the smaller circle. If the area of the larger circle is 60.84π, what is the area of the smaller circle?

F. 51.84π
G. 40.96π
H. 31.36π
J. 20.28π
K. 6.76π

37. What is the equation of a line that is perpendicular to and has the same y-intercept as the line $y = \frac{2}{3}x + 3$?

A. $y = \frac{2}{3}x + 3$

B. $y = -\frac{2}{3}x - 3$

C. $y = -\frac{3}{2}x - 3$

D. $y = -\frac{3}{2}x + 3$

E. $y = \frac{3}{2}x - 3$

38. For which real values of x is $\frac{3xy - 2}{|x| - 2}$ defined?

F. All real values
G. All real values except 2
H. All real values except 2 and -2
J. All real values except $\frac{1}{2}$
K. All real values except 0

39. What is the slope of the line passing through points (2,4) and (4,5)?

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $-\frac{1}{2}$

E. $\frac{3}{2}$

40. The formula for calculating voltage of a circuit is $E = IR$, where E = volts, I = amperes, and R = ohms. If a circuit contains 4 amperes and 2.25 ohms, what is the voltage of the circuit?

F. 1.8 volts
G. 4 volts
H. 6 volts
J. 9 volts
K. 12 volts

41. What figure will be created by connecting each of the following points with a line segment?

$P\ (-2,-2)$ $Q\ (2,4)$ $R\ (6,10)$

A. A line segment
B. An equilateral triangle
C. An isosceles triangle
D. A circle
E. A scalene triangle

42. $\frac{x^8 + x^6 - x^4}{x^4 + 4x^4} = ?$, where $x \neq 0$.

F. $\frac{x^4 + x^2 + x}{5}$

G. $\frac{x^2 + x^2 + 1}{x + 4}$

H. $\frac{x + 2}{x}$

J. $\frac{x^4 + x^2 - 1}{x^4}$

K. $\frac{x^4 + x^2 - 1}{5}$

43. Jack is twice Beth's age, and Alice, who is 14, is four years older than Beth. How old is Jack?

A. 20
B. 24
C. 28
D. 32
E. 36

44. What is the product of all the solutions to the equation $2x^2 - 7x + 6 = 0$?
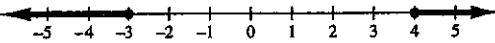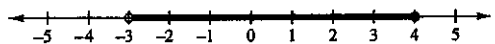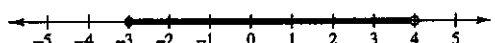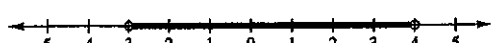
F. $1\frac{1}{2}$

G. 3
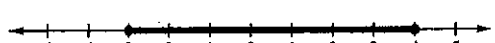
H. $3\frac{1}{2}$

J. 5

K. $5\frac{1}{2}$

GO ON TO THE NEXT PAGE.

Model Mathematics ACT II   413

45. What is the value of $(\cos 30°)(\sec \frac{\pi}{3})$ + $(\csc 60°)(\sin \frac{\pi}{6})$?

A. $\sqrt{3} + \frac{1}{\sqrt{3}}$

B. $\frac{3}{2} + \sqrt{3}$

C. $\frac{2}{3} + \sqrt{3}$

D. $\sqrt{\frac{3}{3}} + \frac{1}{3}$

E. $\frac{1}{3} + \sqrt{3}$

46. A pizzeria sells 12 cheese pizzas and 9 mushroom pizzas for $210.75. If the mushroom pizzas cost $10.75 each, how much does each cheese pizza cost?

F. $8.75
G. $9.25
H. $9.50
J. $10.00
K. $10.25

47. Which of the following number lines shows the solution to the inequality $(x - 4)(-x - 3) \leq 0$?

A.

B.

C.

D.

E.

48. In $\triangle XYZ$ in the diagram below, $XY = 1$, $YZ = 1$, and $XZ = \sqrt{2}$. What is the measure of $\angle X$?
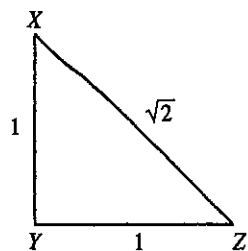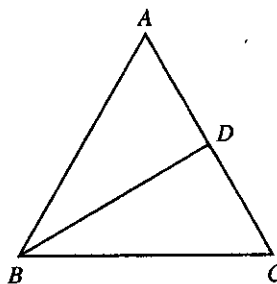

F. 30°
G. 45°
H. 60°
J. 90°
K. 100°

49. If $\tan \theta = \frac{1}{2}$, and the measure of $\angle \theta$ is between 0° and 180°, what is sec θ?

A. $\sqrt{5}$

B. $\frac{4}{8}$

C. 2

D. $\frac{\sqrt{5}}{5}$

E. $\frac{\sqrt{5}}{2}$

50. Simplify: $\frac{(a^2 - b^2)}{2(a + b) + (a + b)} = ?$ $(a > 0, b > 0)$

F. $\frac{a + b}{3}$

G. $\frac{a + b}{a - b}$

H. $\frac{a - b}{3}$

J. $\frac{b - a}{3}$

K. $\frac{a - b}{2(a + b)}$

51. Triangle ABC below is an equilateral triangle and side $\overline{AB}$ is 6 units long. $\overline{BD}$ is a perpendicular bisector of $\overline{AC}$. What is the height of triangle ABC?


A. 2 units
B. $2\sqrt{3}$ units
C. 3 units
D. $3\sqrt{3}$ units
E. 4 units

52. A line in the standard (x,y) coordinate plane contains the point (5,3) and has a slope of $-\frac{2}{3}$. Which of the following points lies on this line?

F. (2,5)
G. (−6,3)
H. (3,4)
J. (−2,−5)
K. (3,5)

GO ON TO THE NEXT PAGE.

53. Jonathan places the bottom of a ladder 9 feet from a wall, as shown in the diagram below. The ladder forms a 50° angle with the ground and touches the wall 15 feet above the ground. Using another ladder, Jonathan again forms a 50° angle with the ground. This ladder touches the wall 9 feet above the ground. How far away from the wall is the bottom of the second ladder?

15 feet

50°

9 feet

9 feet

50°

x
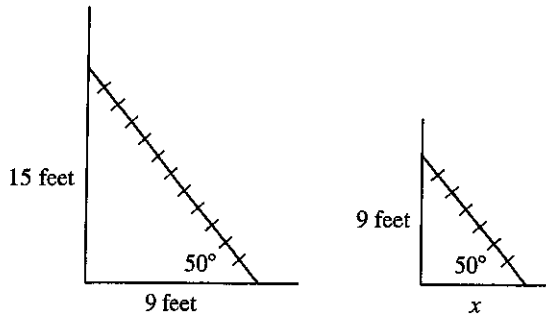
- **A.** 3 feet
- **B.** 4.5 feet
- **C.** 5.4 feet
- **D.** 6 feet
- **E.** 6.2 feet

54. Which of the following is the solution to

$$\frac{|6 - x|}{5} < 4?$$

- **F.** $-26 < x < -14$
- **G.** $14 < x < -26$
- **H.** $-26 < x < 14$
- **J.** $26 < x < -14$
- **K.** $-14 < x < 26$

55. In the figure below, the lengths of $\overline{AD}$, $\overline{DC}$, and $\overline{BC}$ are given in units. What is the area of $\triangle ABC$ in square units?
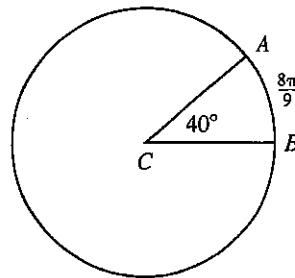
A

3

D

4

C

5

B
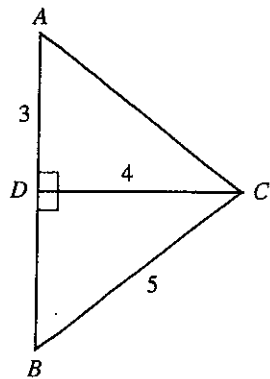
- **A.** 6
- **B.** 12
- **C.** 15
- **D.** 17.5
- **E.** 20

56. Which of the following represents $(7 + 3i)(6 - 4i) + \sqrt{-25}$ in standard form?

- **F.** $42 + 23i$
- **G.** $54 - 5i$
- **H.** $84 + 18i$
- **J.** $5i^2 + 18i + 72$
- **K.** $12i^2 - 5i + 42$

57. In the figure shown, the length of the minor arc $\overarc{AB}$ is $\frac{8\pi}{9}$. The measure of $\angle ACB$ is 40° and point C is the center of the circle. What is the radius of this circle?

A

$\frac{8\pi}{9}$

40°

B

C

- **A.** 4
- **B.** 5
- **C.** 6
- **D.** 7
- **E.** 8

58. Which of the following statements is true for the slopes m_1 and m_2 of lines ℓ_1 and ℓ_2 in the diagram below?

ℓ_1

ℓ_2
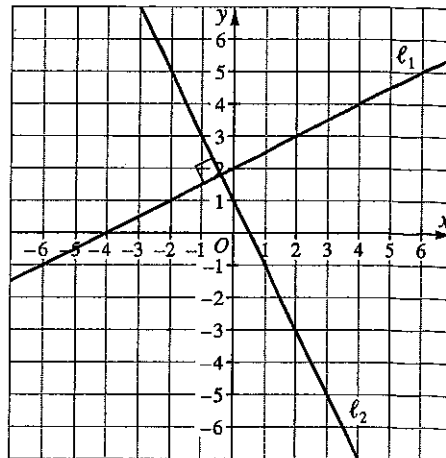
- **F.** $m_1 = m_2$
- **G.** $m_1 \times m_2 = 0$
- **H.** $m_1 \times m_2 = -1$
- **J.** $m_1 = \frac{1}{m_2}$
- **K.** $m_1 = -1m_2$

59. In the right triangle XYZ shown below, the measure of XY is 6 and the measure of YZ is 8. What is the cosine of $\angle Z$?
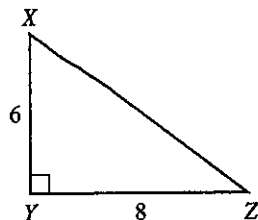
- **A.** $\dfrac{4}{5}$
- **B.** $\dfrac{3}{5}$
- **C.** $\dfrac{5}{3}$
- **D.** $\dfrac{3}{4}$
- **E.** $\dfrac{4}{3}$

60. The line $x = 4$ is graphed on a coordinate plane. Which equation represents a line perpendicular to the line $x = 4$?

- **F.** $x = 2$
- **G.** $y = 4x$
- **H.** $y = 2$
- **J.** $y = 2x - 4$
- **K.** $x - 2y + 6 = 0$

END OF TEST 2

Iodel Mathematics ACT II

ANSWERS

1. A	16. H	31. C	46. H
2. H	17. D	32. J	47. A
3. D	18. J	33. C	48. G
4. H	19. B	34. F	49. E
5. D	20. J	35. B	50. H
6. K	21. E	36. K	51. D
7. C	22. H	37. D	52. F
8. G	23. D	38. H	53. C
9. C	24. H	39. A	54. K
10. J	25. B	40. J	55. B
11. B	26. G	41. A	56. G
12. H	27. B	42. K	57. A
13. C	28. K	43. A	58. H
14. J	29. E	44. G	59. A
15. A	30. K	45. A	60. H

ACT II MATHEMATICS Answers Explained

1. A. Two of the angles in an isosceles triangle have the same angle measure. Angle B cannot have a measure of 110° because the total for the angle measures for the triangle would be more than 180°. So angles A and B must have the same angle measure.

$180° - 110° = 70°$ (the total measure of angles A and B)

$70° \div 2 = 35°$ (the measure of angle B)

2. H. Think: Rent − amount saved = amount needed

$250 − $160 = $90

Mary earns the same amount of money on each of three days because she works the same number of hours. Divide $90 by 3.

$90 ÷ 3 = $30 (amount needed each day)

Mary earns $6 an hour. Divide $30 by $6 to find the number of hours she needs to work to earn $30 a day.

$30 ÷ $6/hour = 5 hours

Mary needs to work 5 hours each day.

3. D. If Basil works 8 hours, he earns

($6)(8) = $48 (regular earnings).

Since Basil earned $84, we know he worked overtime. Find the amount of overtime pay.

$84 − $48 = $36 (overtime earnings)

Find the number of overtime hours.

$36 ÷ $9/hour = 4 hours
(overtime earnings) (overtime pay)

Basil worked 4 overtime hours and 8 regular hours.

8 + 4 = 12

Basil worked 12 hours on Tuesday.

4. H. distance = rate × time

$d = rt$

$260 = 4r$

$r = 65$ mph

5. D. The sum of the measures of supplementary angles is 180°. Answers A and C cannot be correct because ∠1 and ∠8, and ∠4 and ∠5 are linear pairs, which are supplementary. Answers B and E cannot be correct because ∠2 and ∠3, and ∠6 and ∠7 are same-side interior angles, which are supplementary. By elimination, the answer must be D. ∠3 and ∠5 are vertical angles, which means that they are congruent. Line t is not perpendicular to l or to p, so these angles do not measure 90°. Therefore, the total of the measures cannot be 180° and they are not supplementary.

6. K. Write each of the numbers in scientific notation.
$$15,000 = 1.5 \times 10^4 \qquad 2,500 = 2.5 \times 10^3$$
Multiply. $(1.5 \times 10^4)(2.5 \times 10^3) =$
$$(1.5 \times 2.5)(10^4 \times 10^3) = 3.75 \times 10^7$$

7. C. Write the equation. $0.04 \times n = 3.626$
$$n = 3.626 \div 0.04 = 90.65$$
Solve for n.

8. G. Factor the numerator.
$$x^2 - 9 = (x + 3)(x - 3)$$
Substitute $(x + 3)(x - 3)$ for $x^2 - 9$ and simplify.
$$\frac{x^2 - 9}{x - 3} = \frac{(x + 3)(x - 3)}{(x - 3)} = (x + 3)$$

9. C. Substitute 3 for y.
$$3y^2 - 2y + 3$$
$$= 3(3)^2 - 2(3) + 3$$
$$= 3(9) - 6 + 3$$
$$= 27 - 6 + 3$$
$$= 24$$

10. J. Since the sweater was reduced by 15% of the original price, the sweater now costs 85% of the original price $(0.85p)$. Write an equation to find the original price (p).
$$0.85p = \$38.25$$
$$p = \frac{\$38.25}{0.85}$$
$$= \$45.00$$
The original price of the sweater was $45.

11. B. These are the factors for the difference of cubes $(x^3 - y^3)$ where $y = 4$.

12. H. The number of miles the car travels in 1 hour:
$$75 \div 2 = 37.5$$
The number of miles the car travels in 5.5 hours:
$$37.5 \times 5.5 = 206.25$$
Alternative solution:
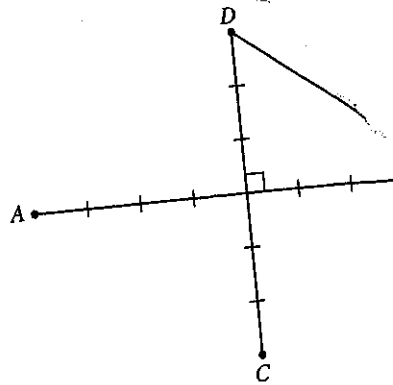Set up a proportion. $\dfrac{\text{miles} \rightarrow}{\text{hours} \rightarrow} \dfrac{75}{2} = \dfrac{x}{5.5}$
Cross multiply and solve.
$$2x = (75)(5.5)$$
$$2x = 412.5$$
$$x = 206.25$$

13. C.

Because line segments $\overline{AB}$ and $\overline{CD}$ are perpendicular bisectors, half of each line segment forms a right triangle with segment $\overline{DB}$. The lengths of the legs of the right triangle are 3 units and 4 units. This must be a 3–4–5 right triangle. Therefore, line segment $\overline{DB}$ has a length of 5.

Alternative solution:
Use the Pythagorean Theorem.
$$c^2 = a^2 + b^2$$
$$f^2 = 3^2 + 4^2$$
$$f^2 = 9 + 16$$
$$f^2 = 25$$
$$f = \pm\sqrt{25}$$
$$f = \pm 5$$
The length of a line segment must be positive, so $f = 5$.

14. J. The formula for the area of a triangle is
$$A = \tfrac{1}{2}bh.$$
Substitute the given values in the formula. The area is 24 square feet. The base is 2 feet less than the height, or $h - 2$.
$$24 = \tfrac{1}{2}(h - 2)h$$
Multiply both sides of the equation by 2.
$$48 = (h - 2)h$$

15. A. First use $I = PRT$ (T is time in years) to find the rate at which this account earns money.
$$I = 3,125 - 2,500 = 625$$
$$I = P \times R \times T$$
$$625 = (2,500)(R)(5)$$
$$625 = 12,500R$$
$$R = 0.05 \text{ or } 5\%$$
Now find the amount of interest $5,000 would earn in 7 years at a rate of 5%. Substitute $5,000 for P, 0.05 for R, and 7 for T.
$$I = PRT$$
$$I = (5,000)(0.05)(7)$$
$$I = 1,750$$
Joan would earn $1,750 in interest.

16. H. To find the area of the rectangular window multiply $l \times w$.

$12 \times 8 = 96$

The formula for the area of a square is
$A = s^2$.

The square has the same area as the rectangle. Write
$s^2 = 96$
$s = \sqrt{96}$
$s = \sqrt{16 \times 6}$
$s = 4\sqrt{6}$ feet

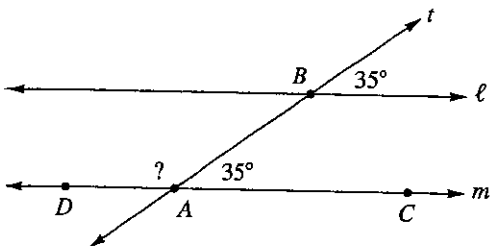The length of a side of the square is $4\sqrt{6}$ feet.

17. D. The number of movies rented the first day is
$0.75 \times 20 = 15$.

The number of movies rented the next day is
$0.90 \times 20 = 18$.

The total number of rentals for both days is
$18 + 15 = 33$.

Therefore, the video store collected $33 \times \$3 = \$99$ total for the two days.

18. J.

Line t is a transversal that crosses parallel lines. $\angle 1$ and $\angle BAC$ are corresponding angles and therefore are congruent. Since the measure of $\angle 1$ is 35°, the measure of $\angle BAC$ is also 35°. $\angle BAC$ and $\angle BAD$ are supplementary, so the measure of $\angle BAD + 35° = 180°$. The measure of $\angle BAD$ is 145°.

19. B. Solve simultaneous equations.

$2x + 3y = 4$ Multiply by 5. $10x + 15y = 20$
$9x + 7.5y = 18$ Multiply by -2. $\underline{-18x - 15y = -36}$
           Add. $-8x = -16$
                          $x = 2$

20. J. Use the order of operations.

$\left(\dfrac{3}{5} + \dfrac{3}{15}\right) \times 5 - \left(\dfrac{3}{4} \div \dfrac{1}{4}\right)$

$= \left(\dfrac{9}{15} + \dfrac{3}{15}\right) \times 5 - \left(\dfrac{3}{4} \div \dfrac{1}{4}\right)$ Rewrite with common denominators.

$= \left(\dfrac{12}{15}\right) \times 5 - \left(\dfrac{3}{4} \times \dfrac{4}{1}\right)$ Work within the parentheses first.

$= \left(\dfrac{4}{5}\right) \times 5 - (3)$ Multiply, then subtract.

$= 4 - 3 = 1$

21. E. Simplify the expression.

$-(3x + 5) - 2x + 2x(2x + 3)$ Use the distributive property.

$= -3x - 5 - 2x + 4x^2 + 6x$
$= 4x^2 + 6x - 3x - 2x - 5$ Combine like terms.

$= 4x^2 + x - 5$

22. H.

$\dfrac{c^2 + 6c + 9}{2c + 6}$

$= \dfrac{(c + 3)(c + 3)}{2(c + 3)}$ Factor the numerator and denominator.

$= \dfrac{(c + 3)}{2}$ Simplify.

23. D. Substitute 7 for x. Simplify the equation and solve for y.
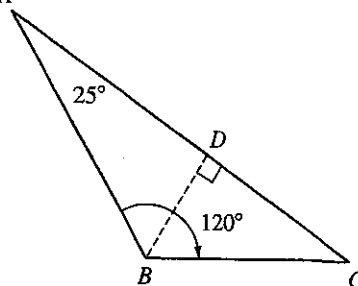
$3y + 4 - 2x = 0$
$3y + 4 - 2(7) = 0$
$3y + 4 - 14 = 0$
$3y - 10 = 0$
$3y = 10$
$y = 3\dfrac{1}{3}$

24. H.

Since $\overline{BD}$ is perpendicular to $\overline{AC}$, the measure of $\angle BDA$ is 90°. We know that the measure of $\angle DAB$ is 25°. The sum of the angle measures in a triangle is 180°, so 25° + 90° + the measure of $\angle ABD$ is 180°. The measure of $\angle ABD$ is 65°. We know that the measure of $\angle ABC$ is 120°, so the measure of $\angle DBC + 65°$ is 120°. Therefore, the measure of $\angle DBC$ is 55°.

25. B. Use factorial to find the number of permutations.

$n! = 6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

There are 720 different orders in which the runners can finish the race.

26. G. The important words in this problem are *is always true*. That means that if you think of even one example in which an answer choice is not true, then that choice is eliminated. Test the answer choices by substituting values for m, n, and p. Let $m = 5$, $n = 4$, and $p = 3$.
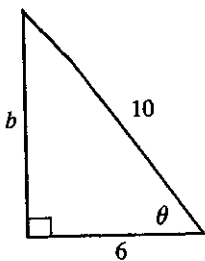
Choice F. $m + n < p$ $5 + 4 < 3$ is false.
Choice G. $2p + m > n$ $2(3) + 5 > 4$
                              $11 > 4$ is true.

G is always true. Since $m > n$, any positive number added to m will also be greater than n.

27. B.

Since cosine $\theta = \dfrac{6}{10}$, we know that the length of leg a is 6 and the length of the hypotenuse is 10. Use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$(6)^2 + b^2 = 10^2$$
$$36 + b^2 = 100$$
$$b^2 = 100 - 36$$
$$b^2 = 64$$
$$b = \pm\sqrt{64}$$
$$b = \pm 8$$

The length of a side must be positive, so $b = 8$.

28. K. The fractions form a proportion. Cross multiply and solve the resulting quadratic equation to find the value of y.

$$\dfrac{2y + 3}{\dfrac{y}{3} + 2} = \dfrac{3y}{6 + y}$$

$(2y + 3)(6 + y) = \left(\dfrac{y}{3} + 2\right)(3y)$ Cross multiply.

$12y + 2y^2 + 18 + 3y = 6y + y^2$ Multiply.

$2y^2 + 15y + 18 = 6y + y^2$ Combine like terms.

$y^2 + 9y + 18 = 0$ Write in quadratic form.

$(y + 3)(y + 6) = 0$ Factor to solve.

$y = -3 \qquad y = -6$

$y = \{-3, -6\}$

29. E. $\angle ABC$ and $\angle FCB$ are congruent alternate interior angles. So, the measure of $\angle FCB$ is also 30°. $\angle FCB$ and $\angle FCH$ are supplementary; the sum of their measures is 180°.

$$30° + \angle FCH = 180°$$
$$\angle FCH = 150°$$

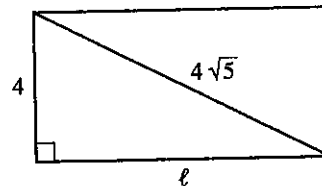**30. K.** Solve. Remember that the absolute value of a number is always positive.

$$2|-8| + 3|-6| - |-2|$$
$$= (2)(8) + (3)(6) - 2$$
$$= 16 + 18 - 2$$
$$= 32$$

31. C.

$$x - 6 = -x^2$$
$$x^2 + x - 6 = 0$$
$$(x - 2)(x + 3) = 0 \quad \text{Factor.}$$
$$x = 2 \qquad x = -3$$
$$x = \{2, -3\}$$

32. J.

Use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

$4^2 + \ell^2 = (4\sqrt{5})^2$, where ℓ is the length of the rectangle

$$16 + \ell^2 = 80$$
$$\ell^2 = 64$$
$$\ell = \pm\sqrt{64}$$
$$\ell = \pm 8$$

The length must be a positive number, so the length is 8.

33. C. The standard equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$. To find the answer to this question, solve for r.

$$r^2 = 54$$
$$r = \pm\sqrt{54} = \pm\sqrt{9 \times 6}$$
$$r = \pm 3\sqrt{6}$$

The radius of a circle must be positive, so $r = 3\sqrt{6}$.

34. F. The slope-intercept equation of a line is $y = mx + b$, where x and y show the location on the coordinate plane, m is the slope, and b is the y-intercept. Substitute the given values for m and the y-intercept. The slope-intercept equation of this line is $y = -2x + 5$.

35. B. Multiply each score by its frequency and add.

$$90 \times 4 = \quad 360$$
$$85 \times 6 = \quad 510$$
$$80 \times 2 = \quad 160$$
$$75 \times 1 = \quad \underline{75}$$
$$\qquad\qquad\quad 1{,}105$$

To find the average score, divide the sum of the scores by the number of scores.

$$1{,}105 \div 13 = 85$$

36. K. First find the radius of the larger circle. The formula for the area of a circle is $A = \pi r^2$. The area of the larger circle is 60.84π, so we can say

$$A = \pi r^2$$
$$60.84\pi = \pi r^2$$
$$60.84 = r^2$$
$$\pm\sqrt{60.84} = r$$
$$r = \pm 7.8$$

A radius has to be positive, so the radius of the larger circle is 7.8. Divide by 3 to find the radius of the smaller circle.

$$7.8 \div 3 = 2.6$$

Find the area of the smaller circle.

$$A = \pi r^2$$
$$A = (2.6)^2\pi$$
$$A = 6.76\pi$$

Alternative solution:

Radius of the larger circle $= r$

Radius of the smaller circle $= \dfrac{r}{3}$

Area of the smaller circle $\left(\dfrac{r}{3}\right)^2\pi = \dfrac{1}{9}r^2\pi$

The area of the smaller circle is $\dfrac{1}{9}$ the area of the larger circle or $\dfrac{60.84\pi}{9} = 6.76\pi$.

37. D. The product of the slopes of perpendicular lines is -1. In other words, the slope of one line is the negative reciprocal of the other. Since the slope of $y = \dfrac{2}{3}x + 3$ is $\dfrac{2}{3}$, the slope of the line perpendicular to it is $-\dfrac{3}{2}$. The y-intercept is the same for both lines, 3. So the equation of the perpendicular line is $y = -\dfrac{3}{2}x + 3$.

38. H. $\dfrac{3xy - 2}{|x| - 2}$ is defined except when $|x| - 2 = 0$.

$|x| - 2 = 0$ when $x = 2$ or when $x = -2$
($|2| = 2$ and $|-2| = 2$)

The expression is defined for all real values of x except 2 and -2.

39. A. The slope is the change in y divided by the change in x.

Use the equation $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to find the slope of the line.

$$m = \dfrac{y_2 - y_1}{x_2 - x_1}$$
$$m = \dfrac{5 - 4}{4 - 2}$$
$$m = \dfrac{1}{2}$$

40. J. Use $E = IR$. Substitute $I = 4$ and $R = 2.25$.
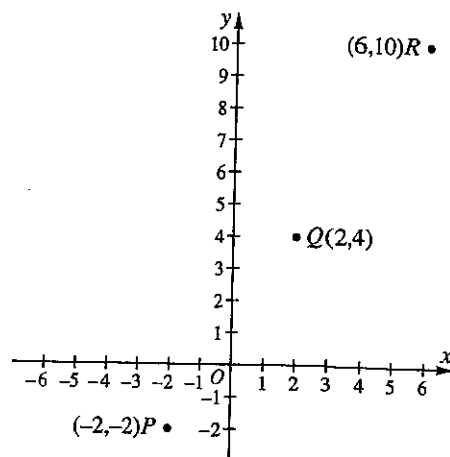
$$E = IR$$
$$E = 4 \times 2.25$$
$$E = 9$$

The voltage in the circuit is 9 volts.

41. A.

The figure cannot be a circle. If the points are on the same line, the figure is a line segment. If the points are not on the same line, the figure is a triangle and we will have to figure out what kind of triangle it is.

Equations for the same line have the same slope. From P to Q: up 6, over 4. From Q to R: up 6 over 4. The slopes are the same, so all three points lie on the same line. Connecting the points forms a line segment.

Alternative solution:

You can also use the slope formula to see if the points are on the same line. Check the slope for the segments $\overline{PQ}$ and $\overline{QR}$. (You can also check the slopes for $\overline{PQ}$ and $\overline{PR}$ or $\overline{QR}$ and $\overline{PR}$.)

$$m = \dfrac{y_2 - y_1}{x_2 - x_1}$$

Slope of $\overline{PQ} = \dfrac{4 - (-2)}{2 - (-2)} = \dfrac{6}{4} = \dfrac{3}{2}$

Slope of $\overline{QR} = \dfrac{10 - 4}{6 - 2} = \dfrac{6}{4} = \dfrac{3}{2}$

The slopes are the same, so all three points lie on the same line. Connecting the points forms a line segment.

You could also *carefully* sketch the three points to find the answer. Remember, though, that a slight sketching error could lead to the wrong answer.

42. K. Simplify. $\dfrac{x^8 + x^6 - x^4}{x^4 + 4x^4} = \dfrac{x^8 + x^6 - x^4}{5x^4}$

$$= \dfrac{x^4(x^4 + x^2 - 1)}{5x^4}$$
$$= \dfrac{x^4 + x^2 - 1}{5}$$

43. A. Write equations to fit the descriptions in the problem. Let $J =$ Jack's age, $A =$ Alice's age, and $B =$ Beth's age.

$J = 2B$ Jack is twice Beth's age.

$A = 14$ Alice is 14.

$A = B + 4$ Alice is 4 years older than Beth.

Since both the second and third equations are equal to A, we can set them equal to each other and solve for B.

$B + 4 = 14$

$B = 10$

To find Jack's age, substitute 10 for B in the equation for Jack's age.

$J = 2B$

$J = 2(10) = 20$

Jack is 20 years old.

44. G. Solve the quadratic equation by factoring.

$2x^2 - 7x + 6 = 0$

$(2x - 3)(x - 2) = 0$

$2x - 3 = 0$ $\qquad x - 2 = 0$

$2x = 3$ $\qquad\qquad x = 2$

$x = \dfrac{3}{2}$

Multiply the two solutions.

$\dfrac{3}{2} \cdot 2 = 3$

45. A.

$(\cos 30°)(\sec \frac{\pi}{3}) + (\csc 60°)(\sin \frac{\pi}{6})$

$= \left(\dfrac{\sqrt{3}}{2} \times 2\right) + \left(\dfrac{2}{\sqrt{3}} \times \dfrac{1}{2}\right)$

$= \dfrac{2\sqrt{3}}{2} + \dfrac{2}{2\sqrt{3}}$

$= \dfrac{\sqrt{3}}{1} + \dfrac{1}{\sqrt{3}}$

$= \sqrt{3} + \dfrac{1}{\sqrt{3}}$

46. H. Write an equation to find the cost of a cheese pizza. Use x to represent the cost of a cheese pizza.

$12x + 9(\$10.75) = \$210.75$

$12x + \$96.75 \quad = \$210.75$

$12x \qquad\qquad = \$114$

$x \qquad\qquad\quad = \$9.50$

47. A. Solve the inequality $(x - 4)(-x - 3) \le 0$.

In order for the product to be less than or equal to zero, either one factor must equal zero or one factor must be positive and one negative. (Remember, a positive times a negative is a negative.)

$(x - 4)(-x - 3) \le 0$ when

$x - 4 \le 0$ and $-x - 3 \ge 0$ OR $x - 4 \ge 0$ and $-x - 3 \le 0$

$\quad x \le 4$ and $\qquad x \le -3$ OR $\quad x \ge 4$ and $\qquad x \ge -3$

$\qquad\quad x \le -3$ $\qquad\qquad$ OR $\qquad\qquad x \ge 4$

x can be less than or equal to -3 or x can be greater than or equal to 4. This inequality matches choice A.

48. G. You may recognize that $\triangle XYZ$ is a 45–45–90 triangle. Since $\overline{XZ}$ is the longest side, the measure of $\angle Y$ is 90°. The measures of $\angle X$ and $\angle Z$ must then be 45°.

Alternative solution:

Use the Pythagorean Theorem, $a^2 + b^2 = c^2$, to determine that $\triangle XYZ$ is a right triangle.

Since $\overline{XY}$ and $\overline{YZ}$ are the same length, the measures of $\angle X$ and $\angle Z$ will be equal.

$90° + 2(m\angle X) = 180°$

$m\angle X = 45°$

49. E. Recall that

$\tan \theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{1}{2}$

$\sec \theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{c}{2}$

Use the Pythagorean Theorem to find the length of the hypotenuse. We know that the legs have lengths 1 and 2.

$c^2 = a^2 + b^2$

$c^2 = 1^2 + 2^2$

$c^2 = 5$

$c = \pm\sqrt{5}$

You know that θ is between 0° and 180° and the tangent is positive. This means θ is between 0° and 90°, so the secant is positive. Therefore, $c = +\sqrt{5}$.

$\text{secant } \theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{\sqrt{5}}{2}$.

Alternative solution:

Use a trigonometric identity and substitute.

$\sec^2 \theta = 1 + \tan^2 \theta$

The tangent of θ is $\dfrac{1}{2}$.

$\sec^2 \theta = 1 + \left(\dfrac{1}{2}\right)^2$

$\sec^2 \theta = 1 + \dfrac{1}{4} = \dfrac{5}{4}$

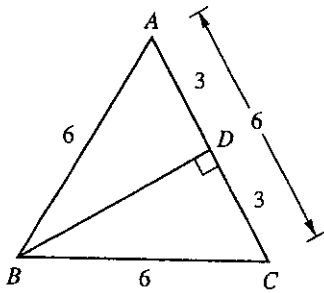$\sec \theta = \sqrt{\dfrac{5}{4}} = \dfrac{\sqrt{5}}{2}$

50. H. Factor and simplify.

$\dfrac{(a^2 - b^2)}{2(a + b) + (a + b)}$

$= \dfrac{(a + b)(a - b)}{3(a + b)}$

$= \dfrac{a - b}{3}$

51. D.

Each side of the triangle has a length of 6 units. Since $\overline{BD}$ is a perpendicular bisector of $\overline{AC}$, the length of $\overline{DC}$ is 3 units and $\angle BDC$ is 90°. We have formed a new triangle BDC, which is a right triangle. Use the Pythagorean Theorem to find the height of the triangle.

$$(DB)^2 + (DC)^2 = (BC)^2$$
$$(DB)^2 + 3^2 = 6^2$$
$$(DB)^2 = 6^2 - 3^2$$
$$(DB)^2 = 36 - 9$$
$$(DB)^2 = 27$$
$$DB = \sqrt{27}$$
$$DB = 3\sqrt{3}$$

52. F. The slope is the change in y divided by the change in x. Try each of the answer choices until you find a slope of $-\frac{2}{3}$.

Choice F:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{3 - 5}{5 - 2} = -\frac{2}{3}$$

A line passing through (5,3) and (2,5) has a slope of $-\frac{2}{3}$.

53. C. Since the wall and the ground form a 90° angle in both figures, and both ladders form a 50° angle with the ground, the angles formed by both ladders at the wall each measure 40° (90° + 50° + 40° = 180°), so the two triangles are similar. In similar triangles, the ratio of corresponding sides is proportional. Write a proportion to represent the height and distance from the wall of each triangle and solve for x.

$$\frac{15}{9} = \frac{9}{x}$$

$15x = 81$ Cross multiply.
$x = 5.4$ Solve.

54. K. Solve. $\dfrac{|6 - x|}{5} < 4$

$$|6 - x| < 20$$

$6 - x < 20$ and $6 - x > -20$
   $-x < 14$ and $-x > -26$
    $x > -14$ and $x < 26$
$$-14 < x < 26$$

55. B. The two smaller triangles are congruent because they are both 3–4–5 right triangles. Find the area of one of the smaller triangles.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2} \times 3 \times 4$$
$$A = 6$$

Since there are two triangles, $6 \times 2 = 12$.

Alternative solution:

Add the bases of the smaller triangles and find the area of the larger triangle.

$$AD + DB = 3 + 3 = 6$$
$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2} \times 6 \times 4$$
$$A = 12$$

56. G. Remember: $i^2 = -1$

$$(7 + 3i)(6 - 4i) + \sqrt{-25}$$
$$= 42 - 28i + 18i - 12i^2 + \sqrt{-25}$$
$$= 42 - 28i + 18i - 12i^2 + 5i$$
$$= 42 - 10i - 12(-1) + 5i$$
$$= 42 - 10i + 12 + 5i$$
$$= 54 - 5i$$

57. A. The circumference of a circle is $2\pi r$. Since 40° is one-ninth of 360°, the length of arc $\widehat{AB}$ is $\frac{1}{9}$ of the circumference, or $\frac{1}{9}(2\pi r)$. We know that the length of arc $\widehat{AB}$ is $\frac{8\pi}{9}$. Substitute to find the measure of the radius.

$$AB = \frac{1}{9}(2\pi r)$$
$$\frac{8\pi}{9} = \frac{1}{9} \times 2\pi r$$
$$\frac{8\pi}{9} = \frac{2\pi r}{9}$$
$$\frac{8\pi}{2\pi} = r$$
$$4 = r$$

58. H. The product of the slopes of perpendicular lines is -1. Since the lines are perpendicular, $m_1 \times m_2 = -1$.

59. A. Use the Pythagorean Theorem to find the length of $\overline{XZ}$.

$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = (XZ)^2$$
$$36 + 64 = (XZ)^2$$
$$100 = (XZ)^2$$
$$10 = XZ$$

$$\cos Z = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{8}{10} = \frac{4}{5}$$

You might also recognize this as a 6–8–10 right triangle.

60. H. Look at the equation $x = 4$. It means that for any y, x will always be 4. This is a vertical line 4 units to the right of the y-axis. A line perpendicular to a vertical line must be a horizontal line. The only horizontal line given is $y = 2$. This equation means that for any x, y will always be 2.