Web Resources

Compound Interest Lesson:
http://www.mathwarehouse.com/compound-interest/formula-calculate.php

Compound Interest Calculator (Solves for any variable)
http://www.mathwarehouse.com/calculators/online-compound-interest-calculator.php

Exponential Growth Lesson
http://www.mathwarehouse.com/exponential-growth/graph-and-equation.php

Difficult Compound Interest Problems

Formulas:

$\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{nt}}$ $A = ending\ dollar\ amount$ $P = principal, beginning\ dollar\ amount$ $r = interest\ rate\ in\ decimal\ form$ $n = number\ of\ times\ the\ interest\ is\ compounded\ annually$ $(annually = 1, semiannually = 2, quarterly = 4, monthly = 12)$ $t = years$	$\boldsymbol{A = Pe^{rt}}$ $A = ending\ dollar\ amount$ $P = principal, beginning\ dollar\ amount$ $e = constant\ \approx 2.71$ $r = interest\ rate\ in\ decimal\ form$ $t = years$

Example 1:

$A = \$1595.43$ $P = \$1250.00$ $r = ???$ $n = continuous$ $t = 4$	$\boldsymbol{A = Pe^{rt}}$ $1595.43 = 1250.00e^{r\cdot 4}$ $\frac{1595.43}{1250.00} = \frac{\cancel{1250.00}e^{r\cdot 4}}{\cancel{1250.00}},\quad divide\ both\ sides\ by\ 1250.00$ $1.276344 = e^{r\cdot 4}$ $\ln 1.276344 = \ln e^{r\cdot 4},\quad take\ the\ natural \log of\ both\ sides$ $\ln 1.276344 = r \cdot 4 \cdot (\ln e),\ \ the\ exponent\ can\ be\ brought\ down, and\ \ln e\ equals\ 1$ $0.2440 = r \cdot 4$ $\frac{0.2440}{4} = \frac{r \cdot \cancel{4}}{\cancel{4}},\quad divide\ both\ sides\ by\ 4$ $0.06099 = r \approx 6.1\%$

Example 2:

If at the end of six years your savings account has a balance of $1236.34, and your original deposit was $1,000.00, then at what interest rate is your account compounded semi-annually?

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$A = 1236.34$
$P = 1000.00$
$r = ???$
$n = 2$
$t = 6$

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$\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{nt}}$
$1236.34 = 1000\left(1 + \frac{r}{2}\right)^{2 \cdot 6}$
$\frac{1236.34}{1000} = \frac{\cancel{1000}\left(1 + \frac{r}{2}\right)^{12}}{\cancel{1000}}$, *divide both sides by* 1000
$1.23634 = \left(1 + \frac{r}{2}\right)^{12}$, *from this point there are two methods for solving*

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<th>Method A:</th>
<th>Method B:</th>
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$\log(1.23634) = \log\left(1 + \frac{r}{2}\right)^{12}$ *take the* log *of both sides*
$0.0921 = 12 \cdot \log\left(1 + \frac{r}{2}\right)$ *bring exponent down*
$\frac{0.0921}{12} = \frac{\cancel{12} \cdot \log\left(1 + \frac{r}{2}\right)}{\cancel{12}}$ *divide both sides by* 12
$0.007675 = \log_{10}\left(1 + \frac{r}{2}\right)$
$10^{0.007675} = 1 + \frac{r}{2}$ *rewrite equation exponentially*
$1.0178 = 1 + \frac{r}{2}$
$1.0178 - 1 = 1 + \frac{r}{2} - 1$ *subtract* 1 *from both sides*
$0.0178 = \frac{r}{2}$
$2 \cdot 0.0178 = \frac{r}{2} \cdot 2$ *multiply both sides by* 2
$0.0357 = r \approx 3.6\%$

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$(1.23634)^{\frac{1}{12}} = \left(\left(1 + \frac{r}{2}\right)^{12}\right)^{\frac{1}{12}}$ *raise both sides to the* $\frac{1}{12}$ *power*
$1.0178 = 1 + \frac{r}{2}$
$1.0178 - 1 = 1 + \frac{r}{2} - 1$ *subtract* 1 *from both sides*
$0.0178 = \frac{r}{2}$
$2 \cdot 0.0178 = \frac{r}{2} \cdot 2$ *multiply both sides by* 2
$0.0357 = r \approx 3.6\%$

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1. $A = \$590.29, P = \$500.00, r = ???, n = continuous, t = 2$

2. $A = \$590.29, P = \$500.00, r = ???, n = continuous, t = 20$
   What is the connection between the answers in number one and number two?

3. $A = \$34{,}826.26, P = \$18{,}000.00, r = ???, n = continuous, t = 12$

4. $A = \$143.24, P = \$111.00, r = 5.1\%, n = continuous, t = ???$

5. $A = \$578.28, P = \$515.20, r = ???, n = continuous, t = 3.5$

6. $A = \$459.08, P = \$300.00, r = ???, n = 2, t = 10$

7. $A = \$1{,}948.84, P = \$1{,}000.00, r = ???, n = 1, t = 10$

8. $A = \$5{,}024.03, P = \$4{,}728.18, r = ???, n = 12, t = 6\ months\ (0.5\ years)$

9. $A = \$5{,}602.39, P = \$5{,}200.00, r = 5.0\%, n = 4, t = ???$

10. $A = \$1{,}255{,}407.48, P = \$1{,}000{,}000.00, r = ???, n = 4, t = 12$

11. A continuously compounded savings account had an initial deposit of \$10,000.00 and 10 years later has a balance of \$13,125.87. At what interest rate was the savings account?

12. $250.00 is left in a savings account at 4.0% and the interest is compounded continuously. If the balance is now $330.78, then how many years was the money been in the account?

13. Hearing about the PlayStation 4 release 3.5 years ago, a teenager put his savings of $500.00 into a continuously compounded savings account. He now has $619.65. At what fixed rate was the interest?

14. Cailynn, an eight year old girl has saved up a total of $400.00 from birthday checks from her grandparents over the years. Her parents put the money into a savings account for her. For the next two years it is earning interest compounded monthly. When she turns 10 years old she has a balance of $507.89. What is her account's interest rate? How much did the account balance increase?

15. Thomas, Cailynn's older brother, is 16 years old. He has saved $800.00 and his parents put the money in an account exactly the same as Cailynn's. At the end of the two years he has $1,015.79. What is his account's interest rate? How much did the account balance increase?

16. Explain the relationship between the accounts in problems 14 and 15.

17. James has won a relatively small lottery amount of $100,000.00. He has two offers from his bank to choose from to deposit his money. The first offer is for three years, compounded monthly at 6.25%. The second offer is for 15 years, compounded monthly at 1.25%. Calculate the ending amount for both offers. Notice that the interest rate is divided by five here, and the years are multiplied by five. Compare with problems 1 and 2. Why do the offers have different ending balances?

18. Your older sister is about to make you an aunt/uncle. As a gift you deposit $100.00 into an account that compounds interest quarterly. In 50 years, the account has a balance of $347.68. What is the interest rate?

19. Before solving this problem, do you expect a bigger account balance or smaller account balance than problem 18? As a gift you decide deposit $100.00 into an account that compounds interest continuously at 2.5%. What is the account balance after 50 years? Were you correct? Explain the comparison.

20. Maybe you have heard that time is money. If you deposit $10,000.00 into an account that compounds interest quarterly for 40 years, you will have a balance of $211,307.65. What is the interest rate? If you have the chance to put the same deposit in a continuously compounded account at the same interest rate, how much quicker will you get to a balance of $211,307.65?

Answer Key:

1. 8.3%
2. 0.83%, years are multiplied by 10, rate is divided by 10
3. 5.5%
4. 5
5. 3.3%
6. 4.3%
7. 6.9%
8. 12.2%
9. 1.5
10. 1.9%
11. 2.72%
12. 7
13. 6.13%
14. 12.0%, $107.89
15. 12.0%, $215.79
16. Double the money deposited will earn double the interest if all other factors are the same
17. $120,564.35, $120,611.25, monthly compounding interest accumulates slower than continuous
18. 2.5%
19. Bigger due to the more frequent compounding, $349.03
20. 7.7%, It will take 39.6 years, so 0.4 years quicker