

Inverse Functions Study Guide

<p>To find the inverse of a function,</p> <ol style="list-style-type: none"> 1. Switch x and y values 2. Solve for y <p>Inverse notation: $f^{-1}(x)$</p> <p>$x = \frac{y}{2} + 2$ $xy + 2y = 4$ $xy = 4 - 2x$</p>	<p>Find the inverse of each function:</p> <p>$f(x) = 2x^2 - 8$ $f^{-1}(x) = \pm \sqrt{\frac{x+8}{2}}$</p> <p>$f(x) = 4x + 2$ $f^{-1}(x) = \frac{x-2}{4}$</p> <p>$f(x) = \frac{2}{3}x + 6$ $f^{-1}(x) = \frac{3(x-6)}{2}$</p> <p>$f(x) = (x-1)^2 + 4$ $f^{-1}(x) = \sqrt{x-4} + 1$</p> <p>$f(x) = \sqrt{2x-1}$ $f^{-1}(x) = \frac{x^2+1}{2}$</p> <p>$f(x) = \frac{4}{x+2}$ $f^{-1}(x) = \frac{4-2x}{x}$</p>
<p>One to One functions:</p> <p>Functions that pass both the vertical and the horizontal line test</p>	<p>Determine if these functions are one to one:</p> <p>$f(x) = 3x + 4$ <i>yes</i></p> <p>$f(x) = (x+2)^2$ <i>no</i></p> <p>$f(x) = \frac{2}{3}x + 5$ <i>yes</i></p> <p>$f(x) = \sqrt{x-7}$ <i>yes</i></p>
<p>Domain and Range of Inverse Functions</p> <p>Remember that the domain and range of the function are the range and domain of its inverse</p> <p>You have to restrict the domain of the function, to ensure that the inverse is a function as well.</p>	<p>Find the domain and range of the given function and its inverse function. Restrict the domain where necessary.</p> <p>$f(x) = (x-4)^2$ $D = (4, \infty)$ $D_{f^{-1}(x)} = (0, \infty)$ $R = (0, \infty)$ $R_{f^{-1}(x)} = (4, \infty)$</p> <hr/> <p>$f(x) = 6x - 1$ $D_{f^{-1}(x)} = (-\infty, \infty)$ $D = (-\infty, \infty)$ $R_{f^{-1}(x)} = (0, \infty)$ $R = (-\infty, \infty)$</p> <hr/> <p>$f(x) = \sqrt{x+4}$ $D_{f^{-1}(x)} = (0, \infty)$ $D = (-4, \infty)$ $R_{f^{-1}(x)} = (-4, \infty)$ $R = (0, \infty)$</p>

Composite functions

Functions are inverses of each other if

$$f \circ g(x) = g \circ f(x) = x$$

Or $f(g(x)) = (gf(x)) = x$

Verify that the functions are inverses of each other.

$$f(x) = \frac{x+1}{3} \text{ and } g(x) = 3x-1$$

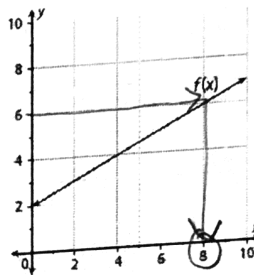
$$f \circ g(x) = \frac{3x-1+1}{3} = x \quad g \circ f(x) = 3\left(\frac{x+1}{3}\right) - 1 = x$$

$$f(n) = \frac{-16+n}{4} \quad f \circ g(x) = \frac{-16+4x+16}{4} = x$$

$$g(n) = 4n+16 \quad g \circ f(x) = 4\left(\frac{-16+n}{4}\right) + 16 = n$$

Inverses from tables and graphs

The graph shows an invertible function $f(x)$. What is the value of $f^{-1}(6)$?



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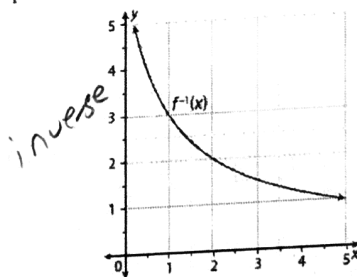
The table shows an invertible function $f(x)$. What is the value of $f^{-1}(4)$?

x	0	1	2	3	4	5
$f(x)$	0	1	4	9	16	25

- a. $\frac{1}{4}$
- b. 2

- c. 4
- d. 16

Use the given graph of the inverse function $f^{-1}(x)$ to find $f(1)$, $f(4)$, $f^{-1}(1)$, and $f^{-1}(3)$.



$$f(1) = 5$$

$$f(4) = 0.5$$

$$f^{-1}(1) = 3$$

$$f^{-1}(3) = 1.5$$

Evaluating Inverses

If g is the inverse of f and $f(9)=5$, what is $f(9) + g(5)$?

Since $f(9)=5$ then $g(5)=9$

$$5 + 9 = 14$$

Using the same values as in the example.

$$\text{Find } 3f(9). = 3(5) = 15$$

$$\text{Find } f(9) - g(5).$$

$$5 - 9 = -4$$