Functions

Study Guide

I. Function Characteristics

Domain: *Interval* of possible <u>x values</u> for a given function. (Left,Right) or as an inequality

Range: *Interval* of possible <u>y values</u> for a given function. (down, up) or as an inequality

End Behavior: What is happening at the far ends of the graph?

For each side	Left side	Right side
	$x \to -\infty$,	$x \to \infty$
Pick one of these	Points Down	Points Up
	$y \to -\infty$	$y \to \infty$

Increasing Intervals: *Interval* of <u>x values</u> for which the corresponding y values are increasing.

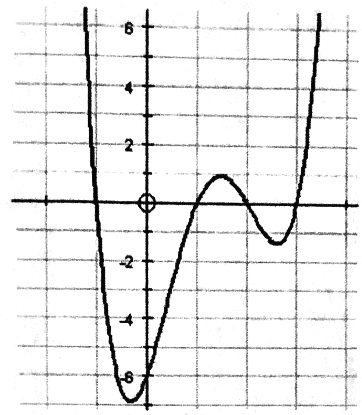
Decreasing Intervals: *Interval* of <u>x values</u> for which the corresponding y values are decreasing.

x-Intercepts/roots/zeroes: *points* where the graph crosses the x axis. $(x, 0)$

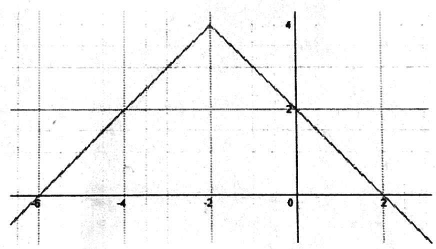
y-Intercepts: *points* where the graph crosses the y axis. $(0, y)$

Maximums: *points* where the graph changes from increasing to decreasing. Peaks in the graph.

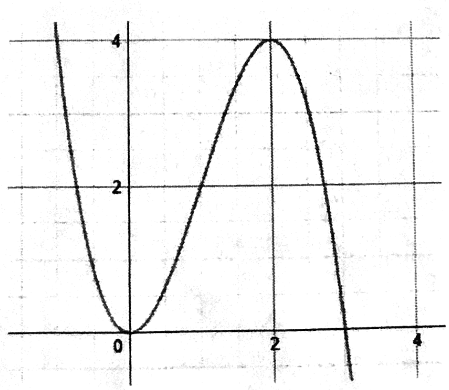
Minimums: *points* where the graph changes from decreasing to increasing. Valleys in the graph.



Domain: $(-\infty, \infty)$
Range: $[-7, \infty)$
End Behavior:
As $x \to -\infty, y \to \infty$
As $x \to \infty, y \to \infty$
Increasing Intervals:
$(-0.5, 1.5), (2.5, \infty)$
Decreasing Intervals:
$(-\infty, -0.5), (1.5, 2.5)$
x-Intercepts:
$(-1,0), (1,0), (2,0), (3,0)$
y-Intercepts: $(0, -6)$
Maximums: $(1.5, 1)$
Minimums:
$(-0.5, -7), (2.5, -1.25)$



Domain: $(-\infty, \infty)$ ℝ
Range: $(-\infty, 4]$ $y \le 4$
End Behavior:
As $x \to -\infty$, $y \to -\infty$
As $x \to \infty$, $y \to -\infty$
Increasing Intervals:
$(-\infty, -2)$
Decreasing Intervals:
$(-2, \infty)$
x-Intercepts: $(-6,0)$ $(2,0)$
y-Intercepts: $(0,2)$
Maximums: $(-2,4)$
Minimums: None



Domain: $(-\infty, \infty)$ ℝ
Range: $(-\infty, \infty)$ ℝ
End Behavior:
As $x \to -\infty$, $y \to \infty$
As $x \to \infty$, $y \to -\infty$
Increasing Intervals:
$(0, 2)$
Decreasing Intervals:
$(-\infty, 0), (2, \infty)$
x-Intercepts: $(0,0)$ $(3,0)$
y-Intercepts: $(0,0)$
Maximums: $(2,4)$
Minimums: $(0,0)$

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II. Function Transformations

General form: $g(x) = a\,f(x-h)+k$

f(x)	parent function
g(x)	transformed function
a	if negative, flip vertically $0<\lvert a\rvert<1$ vertical compression $\lvert a\rvert>1$ vertical stretch
h	if negative, horizontal shift right if positive, horizontal shift left
k	if negative, vertical shift down if positive, vertical shift up

Examples

$E1.\ g(x) = x^2+2$
Parent Function:
quadratic
Transformations:
shift up 2 units

$E2.\ g(x) = -(x-4)^3-1$
Parent Function:
cubic
Transformations:
flip vertically
shift right 4 units
shift down 1 unit

$E3.\ g(x) = 3\sqrt{x+1}-7$
Parent Function:
Radical (square root)
Transformations:
Stretch by a factor of 3
Shift left 1 unit
Shift down 7 units

$E4.\ g(x) = -\frac{1}{2}(x-3)^2+1$
Parent Function:
quadratic
Transformations:
Flip vertically
Compression by a factor of $\frac{1}{2}$
Shift Right 3 units
Shift up 1 unit

1. $g(x) = 2^{x-3}+5$
Parent Function:
exponential
Transformations:
→ 3
↑ 5

2. $g(x) = -(x+7)^2$
Parent Function:
Quadratic
Transformations:
flipped over X
← 7

3. $g(x) = 2\log(x-2)-1$
Parent Function:
Log
Transformations:
Vert Stretched
b fo 2
→ 2
↓ 1

III. Graphing a function from an equation - Example

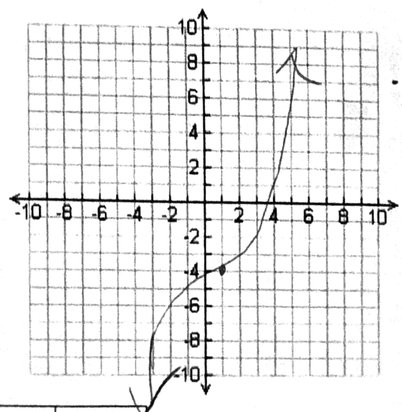
1. Identify the parent function to determine a general shape.
<u>*Cubic*</u>

2. Think about where the vertex or critical points are usually found for the parent function.
<u>*Centered at the origin. Is always increasing from left to right.*</u>

3. Where are the critical points of the new function given the transformations in the equation? <u>*Since there is a horizontal shift right 1 unit and a vertical shift down four units, the center is at the point (1,-4).*</u>

4. Use this information to plan which points to plot on the graph. Make a t table with these points.
<u>*Since the center of the graph is (1,-4), pick two x values on either side of this point and evaluate the f(x) at those x's.*</u>

Graph $f(x) = 2(x-1)^3-4$



x	y
-1	-20
0	-6
1	-4
2	-2
3	12

5. Plot the points and connect the dots.

Graphing a function from an equation

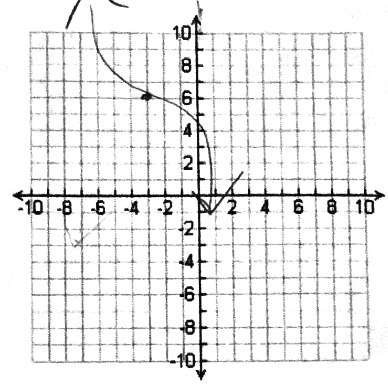
1. Identify the parent function to determine a general shape.

2. Think about where the vertex or critical points are usually found for the parent function.

3. Where are the critical points of the new function given the transformations in the equation?

4. Use this information to plan which points to plot on the graph. Make a t table with these points.

Graph $f(x) = -(x+3)^3 + 6$



x	y

5. Plot the points and connect the dots.

Writing Function Equation from a description of the transformations

How do translations effect equation?

$f(x) = -a(x-h) + k$

"-" flip over x axis

a compression or stretch

h horizontal shift in the opposite direction of the sign

k vertical shift in the same direction of the sign

EXAMPLE

Write the equation for a quadratic function with a vertical shift down 3, left 7 and a vertical stretch by a factor of 4.

Quadratic : x^2

Down 3: -3 from the function (outside)

Left 7: add 7 to x (inside)

V. stretch by 4: multiply by 4

$$y = 4(x+7)^2 - 3$$

Write the equation for an absolute value function that has been compressed by a factor of 2 and shifted down three units

$g(x) = \frac{1}{2}|x| - 3$

Write the equation for a cubic that has been flipped vertically, shifted up 5 units, and shifted right 2 units.

$h(x) = -(x-2)^3 + 5$

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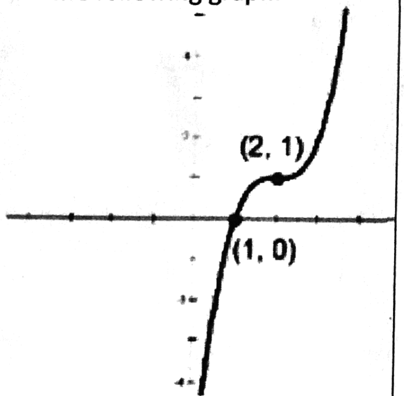
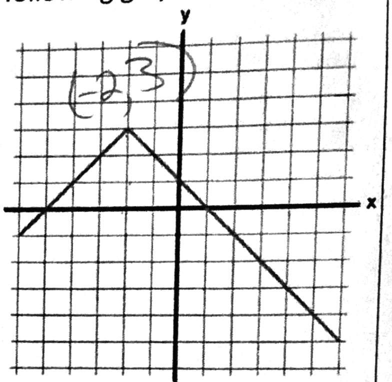
Determining Equation from Graph What's the parent function? Where's the vertex or critical point of the parent function? Where's the vertex or critical point of this function? How did we get from the parent function critical point to the critical point of this function? How do I translate those changes into an equation?	Example: Write the equation for the following graph.  Cubic so x^3 vertex is up 2, right 1 $y = (x-2)^3 + 1$	Write the equation for the following graph  $y = -\|x+2\| + 3$
Shifts of Shifts Apply the stated changes to the appropriate parts of the "starting function".	Example: If the function $f(x) = (x+1)^2 - 1$, what would be the equation of g(x) if g(x) is f(x) shifted left 3 units, up 2 units, flipped vertically and stretched by a factor of 4? Left 3: +3 to x Up 2: + 2 Flipped vertically: - in front Stretched by 4: multiplied by 4 $g(x) = -4(x+1+3)^2 - 1 + 2$ $g(x) = -4(x+4)^2 + 1$	$f(x) = 2(x)^3 + 4$ Find g(x) if g(x) is f(x) shifted up 2, right 1 and compressed by a factor of 6. $g(x) = \frac{1}{3}(x-1)^3 + 6$ $f(x) = -\|x-5\|$ Find g(x) if g(x) is f(x) shifted up 4, left 3, stretched by a factor of 2, and flipped vertically. $g(x) = 2\|x-2\| + 4$
Shifts of Shifts part 2 State the transformations to f(x) that would yield g(x) Example: $f(x) = -3\sqrt{x-4} + 1$ $g(x) = \frac{3}{5}\sqrt{x+4} + 7$ Was +1, now is +7 so went up 6 Was -4 now is +4 so went left 8 Was 3 now 3/5 so compressed by a factor of 5 Was negative, now positive so flipped vertically	$f(x) = \|x+2\| - 3$ $g(x) = -2\|x+1\| + 2$ flipped over x Vert Stretched bfo 2 → 1 ↑ 5	$f(x) = -3(x-1)^2 - 3$ $g(x) = -(x+4)^2 - 5$ Vert Compressed bfo 3 ← 5 ↓ 2

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Function Notation and Function Operations

Function notation uses f(x) instead of y. Plug in whatever number is in the parentheses for x.

Ex. f(x) = 8x+2.
Find f(3)

f(3) = 8(3)+2 = 24 + 2 = 26

Function operations can be indicated like so (f-g)(x) would mean subtract g(x) from f (x). Similarly, (f·g)(2) would mean find the product of f(2) and g(2)

$$f(x) = 2x - 3$$
$$g(x) = -2x^2$$

Find f(3)

$$f(3) = 2(3) - 3$$
$$f(3) = 6 - 3$$
$$f(3) = 3$$

Find f(2) – 2g(1)

$$f(2) = 2(2) - 3$$
$$f(2) = 4 - 3$$
$$f(2) = 1$$

$$g(1) = -2(1)^2$$
$$g(1) = -2(1)$$
$$g(1) = -2$$

f(2) – 2g(1)
1 – 2(-2)
1+4
5

Find (f·g)(2)

$$f(2) = 2(2) - 3$$
$$f(2) = 4 - 3$$
$$f(2) = 1$$

$$g(2) = -2(2)^2$$
$$g(2) = -2(4)$$
$$g(2) = -8$$

$$f(x) = -3x^2$$
$$g(x) = x^2 - 1$$

Find f(-2)

-12

Find 2g(3)+f(-1)

13

Find (g-f)(x)

$4x^2 - 1$

Find (g+f)(1)

-3

(f·g)(-3)

-216

Piecewise Functions

Piecewise Functions behave differently based on the input.

For instance, $f(x) = \begin{cases} 3x, & x < 3 \\ x + 2, & x \geq 3 \end{cases}$

If the input, x, is less than three, use 3x. If it is greater than three, use x+2.

So,

f(4) = 4+2 = 6

f(1) = 3(1) = 3

$$f(x) = \begin{cases} x^2, & x < 1 \\ 2x + 3, & x \geq 1 \end{cases}$$

Find f(0)

0

Find f(1)

5

Find f(2)

7

$$f(x) = \begin{cases} -x, & x \leq 0 \\ 2, & 0 < x < 2 \\ 2x, & x \geq 2 \end{cases}$$

Find f(3) 6

Find f(-5) 5

Findf(1.5) 2

Find f(2) – f(5) + 3f(-6)

12

Find f(-1)+f(0)-f(1)+½f(2)

1

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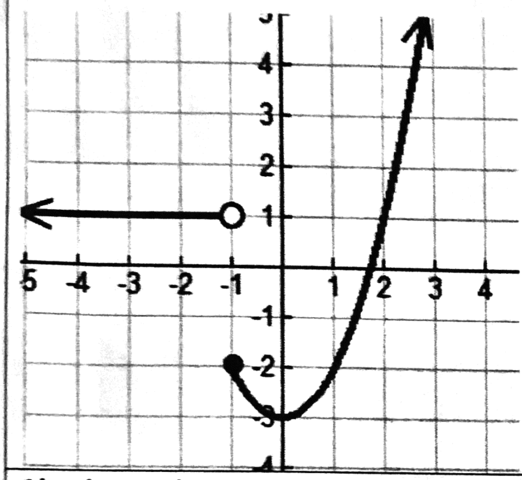
Graphing Piecewise Functions

Graph each section of the graph for and only for its corresponding domain.

So the graph of

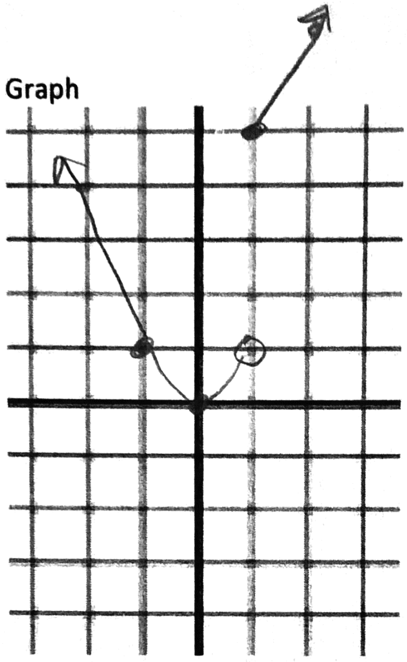
$$f(x) = \begin{cases} 1, & x < -1 \\ x^2 - 3, & x \geq -1 \end{cases}$$

Would be:



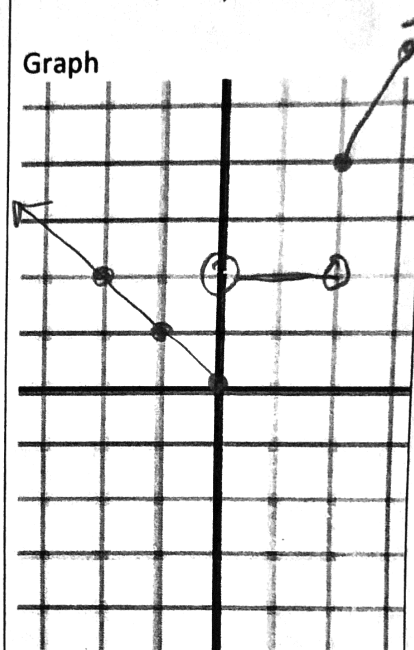
$$f(x) = \begin{cases} x^2, x < 1 \\ 2x + 3, x \geq 1 \end{cases}$$

Graph



$$f(x) = \begin{cases} -x, & x \leq 0 \\ 2, & 0 < x < 2 \\ 2x, & x \geq 2 \end{cases}$$

Graph



Absolute Value Equations and Inequalities

To solve absolute value equations or inequalities:

1. isolate the absolute value
2. drop the bars to create two equations. Change all signs on one side of the second equation.
3. Solve both equations
4. Plug in your answers to the original to check for extraneous solutions.

* For inequalities, don't forget that flipping the signs results in the flipping of the inequality as well.

Tolerance:

$$|x - a| \leq t$$

Where a is the desired amount or the average and t is the tolerance.

$|x - 2| + 4 = 2x + 7$

$x = -\frac{1}{3}$

$|2 - x| < 8$

$-6 < x < 10$

$3|4x - 1| \leq 9$

$-\frac{1}{2} \leq x \leq 1$

An iPhone seven is supposed to weigh 192 grams, but it is allowed a margin of error of up to .4 grams. Write an absolute value inequality to represent this.

$|g - 192| \leq .4$

The ideal weight for a DDR player is 160 pounds, but the game will still perform near optimally for anyone within 85 pounds of that. Write an absolute value inequality to represent this. Solve it.

$|p - 160| \leq 85$

$75 \leq p \leq 245$