Name______________________________ Period ________ Date________________

Unit 3 – Polynomials Study Guide

Objective: Division

Synthetic division can be used when the divisor is in the form (x – k).

Example: Use synthetic division for the following
$(2x^3 - 7x^2 - 8x + 16) \div (x - 4)$

First, write down the coefficients in descending order, and k of the divisor in the form x – k:

$$\begin{array}{r|rrrr} 4 & 2 & -7 & -8 & 16 \\ & & 8 & 4 & -16 \\ \hline & 2 & 1 & -4 & \boxed{0} \end{array}$$

k →

Bring down the first coefficient.

These are the coefficients of the quotient (and the remainder)

Multiply this by k

Add the column.

Repeat the process.

$2x^2 + x - 4$

When _______ is a _________ divide your ________ one more time

Find the quotient and remainder of:

1. $(x^3 + 4x^2 - 3x + 2) \div (x + 3)$

$$\begin{array}{r|rrrr} -3 & 1 & 4 & -3 & 2 \\ & & -3 & -3 & 18 \\ \hline & 1 & 1 & -6 & \boxed{20} \end{array}$$

$\boxed{x^2 + x - 6 + \frac{20}{x+3}}$

2. $(2x^4 - 4x^3 - x^2 - 3x + 8) \div (x - 1)$

$$\begin{array}{r|rrrrr} 1 & 2 & -4 & -1 & -3 & 8 \\ & & 2 & -2 & -3 & -6 \\ \hline & 2 & -2 & -3 & -6 & \boxed{2} \end{array}$$

$\boxed{2x^3 - 2x^2 - 3x - 6 + \frac{2}{x-1}}$

3. $(5x^3 + 3x^2 - 3x - 6) \div (2x + 1)$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 5 & 3 & -3 & -6 \\ & \downarrow & -\frac{5}{2} & -\frac{1}{4} & +\frac{13}{8} \\ \hline & 5 & \frac{1}{2} & -\frac{13}{4} & \boxed{-\frac{35}{8}} \end{array}$$

$6/_2$ $-1^{1}/_{4}$ $-\frac{48}{8}$

$5x^2 + \frac{1}{2}x - \frac{13}{4} - \frac{35}{8(2x+1)}$

If there is a missing term you need to put in a 0

Find the quotient and remainder of:

4. $(x^3 + 6x + 1) \div (x - 3)$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & 6 & 1 \\ & & 3 & 9 & 45 \\ \hline & 1 & 3 & 15 & \boxed{46} \end{array}$$

$x^2 + 3x + 15 + \frac{46}{x-3}$

5. $(2x^4 + 8 - 4x) \div (x + 2)$

$$\begin{array}{r|rrrrr} -2 & 2 & 0 & 0 & -4 & 8 \\ & \downarrow & -4 & 8 & -16 & 40 \\ \hline & 2 & -4 & 8 & -20 & \boxed{48} \end{array}$$

$2x^3 - 4x^2 + 8x - 20 + \frac{48}{x+2}$

Remainder Theorem:
If a polynomial p(x) is divided by the binomial x – a, the remainder obtained is p(a).

So, looking at our example, if $p(x) = x^3 - 4x^2 - 7x + 10$ was divided by x – 2, the remainder can be determined by finding p(2).

$p(x) = x^3 - 4x^2 - 7x + 10$

$p(2) = (2)^3 - 4(2)^2 - 7(2) + 10$

$= 8 - 16 - 14 + 10 \quad = -12$

Or you can _______ in ____________

6. Determine the remainder when $3x^6 - 3$ is divided by x - 2

$3(2)^6 - 3$

$3 \cdot 64 - 3$

$192 - 3$

$\boxed{189}$

Find k first then do division with other root

Suppose $f(x) = x^3 - x^2 + 4x + k$. The remainder of the division of f(x) by (x – 1) is 12. What is the remainder of the division of f(x) by (x + 3)

$\boxed{-40}$

k=8

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 4 & k \\ & & 1 & 0 & 4 \\ \hline & 1 & 0 & 4 & \boxed{12} \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & -1 & 4 & 8 \\ & & -3 & 12 & -48 \\ \hline & 1 & -4 & 16 & \boxed{-40} \end{array}$$

Just follow the pattern to find each

$$x^3 + x^2 + 7x + 30 + \frac{119}{x - 4}$$

B(x) + R(x) p(x)

If the answer is in form $B(x) + \frac{r(x)}{p(x)}$

p(x)= x-4 B(x)= $x^3+x^2+7x+30$ r(x)= 119

Be able to find the missing dimension. Remember that it usually doesn't matter which expression goes where, unless the problem specifically states it.

Steps: 1. Divide
2. Factor the quadratic.

The volume of a box is given by the polynomial $V(x) = -x^3 + 28x^2 - 71x - 100$. The length is represented by the expression $(x - 4)$.
12/13. Find the expressions that represent the height and width of the box.

$$\begin{array}{r|rrrr} 4 & -1 & 28 & -71 & -100 \\ & & -4 & 96 & 100 \\ \hline & -1 & 24 & 25 & \boxed{0} \end{array}$$

$-x^2 + 24x + 25$

$(x+1)(-x+25)$

$\boxed{(x+1)(25-x)}$

Be able to find the highest possible volume for the box. (find the vertex in the realistic domain)

14. Find the max volume of the box.

1874.1

Be able to give the realistic domain and range of the values of x and y.

15/16. What are the realistic domain and range for this problem?

D: $4 \le x \le 25$ R: $0 \le y$ 1874.1

Finding all roots of a function.

To find all roots:

1. Graph the equation to determine the integer roots.
2. Use synthetic division to find the quadratic equation.
3. Solve the quadratic equation by either factoring or using the quadratic formula

17. Find all of the roots for $f(x) = x^3 - 2x^2 - 2x + 12$.

$x = -2;\ 2 \pm \sqrt{2}\,i$

18. $x^4 - 2x^2 + 3x - 2$

$x = -2;\ 1;\ \frac{1 \pm \sqrt{3}\,i}{2}$

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & -2 & 3 & -2 \\ & \downarrow & -2 & 4 & -4 & 2 \\ \hline & 1 & -2 & 2 & -1 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & 2 & -1 \\ & & 1 & -1 & 1 \\ \hline & 1 & -1 & 1 & \boxed{0} \end{array}$$

$x^2 - x + 1$

$\frac{1 \pm \sqrt{1-4(1)(1)}}{2}$

17.

$$\begin{array}{r|rrrr} -2 & 1 & -2 & -2 & 12 \\ & \downarrow & -2 & 8 & -12 \\ \hline & 1 & -4 & 6 & \boxed{0} \end{array}$$

$x^2 - 4x + 6$

$\frac{4 \pm \sqrt{16-4(1)(6)}}{2}$

$\frac{4 \pm 2\sqrt{2}\,i}{2}$

When is the second function greater than the first

$$y = .2(x-3)^2 + 3x + 8$$

$$y = 2^{.5x-6}$$

$x \ge 27.422$

or Zoom out a lot!

What is a polynomial with roots 2 and 8*i*?

$(x-2)(x-8i)(x+8i)$

$(x-2)(x^2+64)$

$\boxed{x^3 - 2x^2 + 64x - 128}$

What is a polynomial with the roots $\frac{4}{3}, 2, \frac{-1}{6}$?

$(3x-4)(x-2)(6x+1)$

$(3x^2 - 10x + 8)(6x+1)$

$\boxed{18x^3 - 57x^2 + 38x + 8}$