Three high-school soccer players practice kicking goals from the points shown in the diagram. All three points are along an arc of a circle. Player A says she is in the best position because the angle of her kicks toward the goal is wider than the angle of the other players' kicks. Do you agree? Explain.

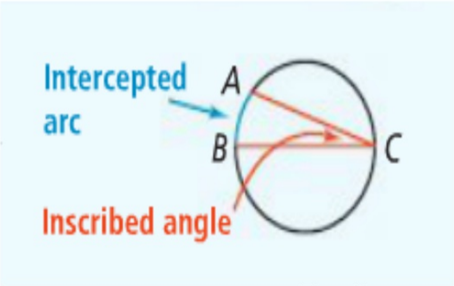
Obj: SWBAT find and use the angle measures of central and inscribed angles.

Agenda:
Warm up
notes
practice
closure

Vocab:

Central angle: an angle whose vertex is at the center of the circle

Inscribed Angle: an angle whose vertex is on the circle and sides are chords of the circle



Intercepted Arc: the arc inside the angle

Key Concept Arc Measure

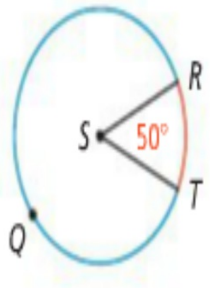
Arc Measure

The measure of a minor arc is equal to the measure of its corresponding central angle.

The measure of a major arc is the measure of the related minor arc subtracted from 360.

The measure of a semicircle is 180.

Example



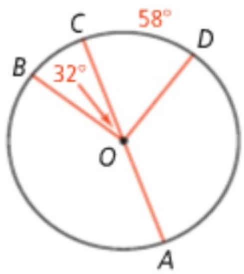
$m\overset{\frown}{RT} = m\angle RST = 50$

$m\overset{\frown}{TQR} = 360 - m\overset{\frown}{RT}$

$= 310$

Problem 2 Finding the Measures of Arcs

What is the measure of each arc in $\odot O$?



A $\widehat{BC}$ $\quad m\widehat{BC} = m\angle BOC = 32$

B $\widehat{BD}$ $\quad m\widehat{BD} = m\widehat{BC} + m\widehat{CD}$

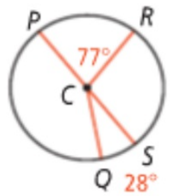
$m\widehat{BD} = 32 + 58 = 90$

C $\widehat{ABC}$ $\quad \widehat{ABC}$ is a semicircle.

$m\widehat{ABC} = 180$

D $\widehat{AB}$ $\quad m\widehat{AB} = 180 - 32 = 148$

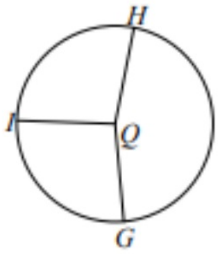
Got It? **2.** What is the measure of each arc in $\odot C$?



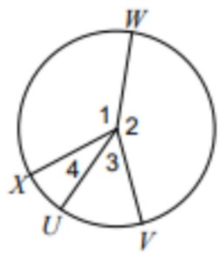
a. $m\widehat{PR}$
b. $m\widehat{RS}$
c. $m\widehat{PRQ}$
d. $m\widehat{PQR}$

Name the major and minor arcs made by the given angle.

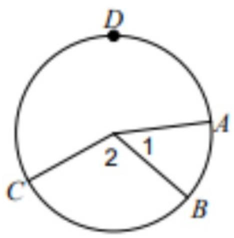
1) $\angle HQG$



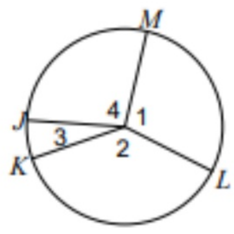
2) $\angle 3$



3) $\angle 1$

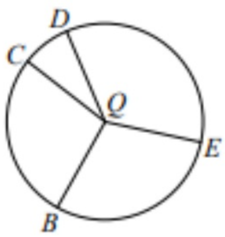


4) $\angle 2$

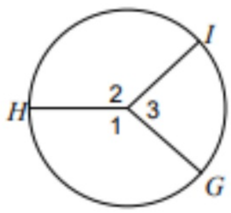


Name the angle that makes the given arc.

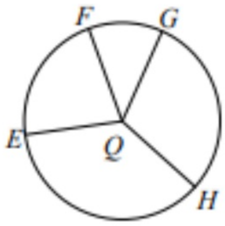
5) $\overset{\frown}{BE}$



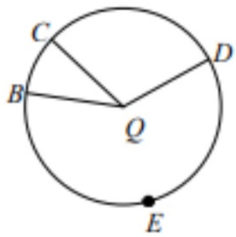
6) $\overset{\frown}{GI}$



7) $\overset{\frown}{EGF}$

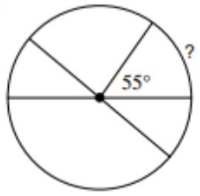


8) $\overset{\frown}{BED}$

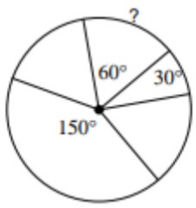


Find the measure of the arc or central angle indicated. Assume that lines which appear to be diameters are actual diameters.

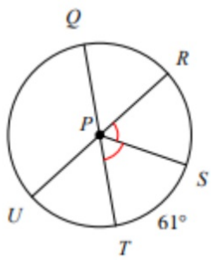
9)



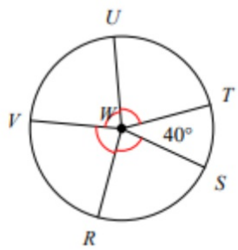
10)



11) $m\angle QPS$

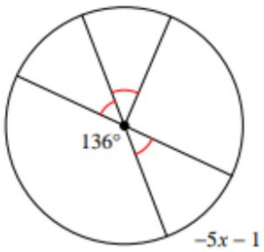


12) $m\angle SWV$

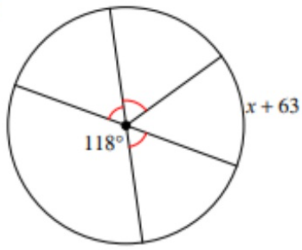


Solve for x. Assume that lines which appear to be diameters are actual diameters.

17)



18)

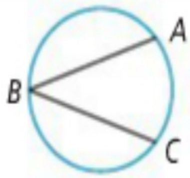


take note

Theorem 12-11 Inscribed Angle Theorem

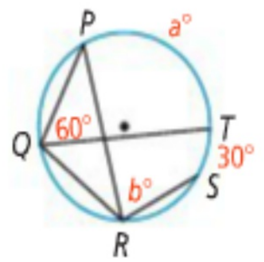
The measure of an inscribed angle is half the measure of its intercepted arc.

$$m\angle B = \frac{1}{2} m\widehat{AC}$$



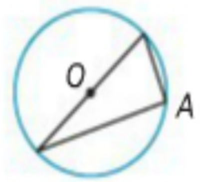
Problem 1 Using the Inscribed Angle Theorem

What are the values of *a* and *b*?

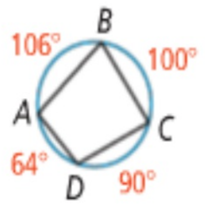


$m\angle PQT = \frac{1}{2} m\widehat{PT}$	Inscribed Angle Theorem
$60 = \frac{1}{2} a$	Substitute.
$120 = a$	Multiply each side by 2.
$m\angle PRS = \frac{1}{2} m\widehat{PS}$	Inscribed Angle Theorem
$m\angle PRS = \frac{1}{2}(m\widehat{PT} + m\widehat{TS})$	Arc Addition Postulate
$b = \frac{1}{2}(120 + 30)$	Substitute.
$b = 75$	Simplify.

Got It? **1. a.** In $\odot O$, what is $m\angle A$?



b. What are $m\angle A$, $m\angle B$, $m\angle C$, and $m\angle D$?

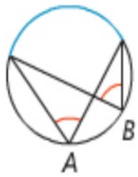


c. What do you notice about the sums of the measures of the opposite angles in the quadrilateral in part (b)?

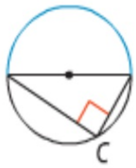
take note

Corollaries to Theorem 12-11: The Inscribed Angle Theorem

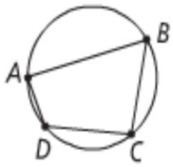
Corollary 1
Two inscribed angles that intercept the same arc are congruent.



Corollary 2
An angle inscribed in a semicircle is a right angle.



Corollary 3
The opposite angles of a quadrilateral inscribed in a circle are supplementary.

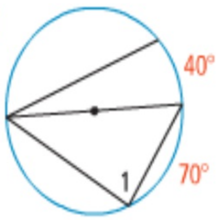


You will prove these corollaries in Exercises 31–33.

Problem 2 Using Corollaries to Find Angle Measures

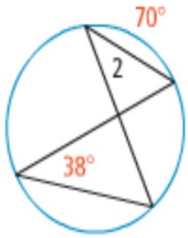
What is the measure of each numbered angle?

A



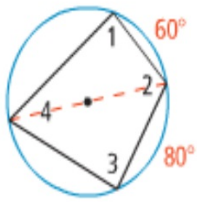
∠1 is inscribed in a semicircle.
By Corollary 2, ∠1 is a right angle, so $m\angle 1 = 90$.

B



∠2 and the 38° angle intercept the same arc. By Corollary 1, the angles are congruent, so $m\angle 2 = 38$.

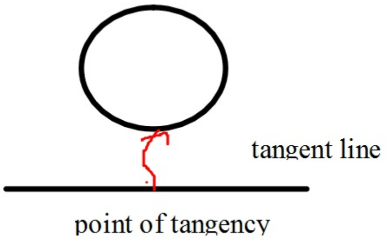
Got It? 2. In the diagram at the right, what is the measure of each numbered angle?

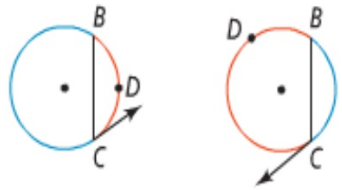


take note

Theorem 12-12

The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.



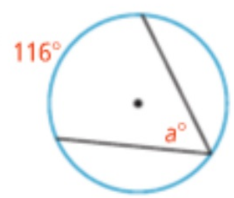


$$m\angle C = \frac{1}{2} m\widehat{BDC}$$

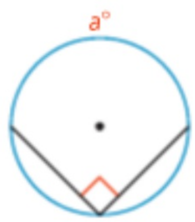
You will prove Theorem 12-12 in Exercise 34.

d the value of each variable. For each circle, the dot represents center.

See Problems 1 and 2.



7.



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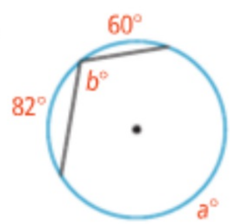

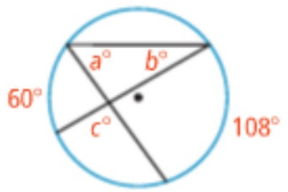

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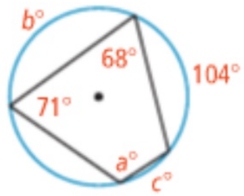

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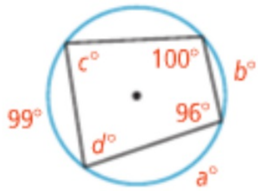
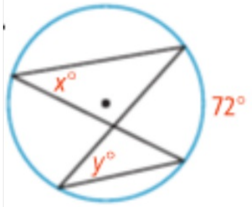
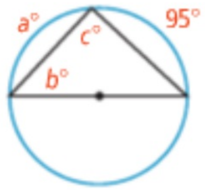
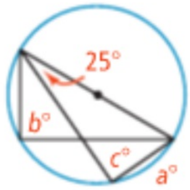




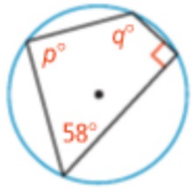
13.



14.

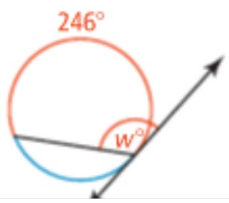


15.



d the value of each variable. Lines that appear to be tangent are tangent.

See Problem 3.



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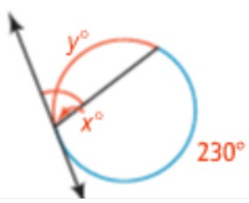

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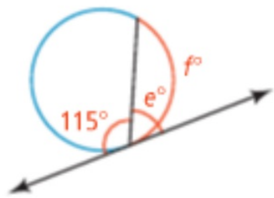

Practice Problems:

Work on the packet.

Closure: What is the relationship between central angles and inscribed angles?